Space-Time Coding for Wireless Communications : Principles and Applications

SPACE-TIME CODING FOR WIRELESS COMMUNICATIONS : PRINCIPLES AND APPLICATIONS

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Kluwer Academic Publishers
Boston/Dordrecht/London

Contents

Acknowledgments

We would like to thank the following individuals (in alphabetical order) for many stimulating discussions and technical contributions to the results presented in this chaper : Sushanta Das, Sanket Dusad, Christina Fragouli, Anastasios Stamoulis, and Waleed Younis.

1. Introduction

The essential feature of wireless transmission is the randomness of the communication channel which leads to random fluctuations in the received signal commonly known as fading. This randomness can be exploited to enhance performance through *diversity*. We broadly define diversity as the method of conveying information through multiple independent instantiations of these random fades. There are several forms of diversity; our focus in this chapter will be on *spatial* diversity through multiple independent transmit/receive antennas. Information theory has been used to show that multiple antennas have the potential to dramatically increase achievable bit rates Telatar, 1999, thus converting wireless channels from narrow to wide data pipes.

The earliest form of spatial transmit diversity is the delay diversity scheme proposed in Uddenfeldt and Raith, 1992; Wittneben, 1993 where a signal is transmitted from one antenna, then delayed one time slot, and transmitted from the other antenna. Signal processing is used at the receiver to decode the superposition of the original and time delayed signals. By viewing multiple-antenna diversity as independent information streams, more sophisticated transmission (coding) schemes can be designed to get closer to theoretical performance limits. Using this approach, we focus on space-time coding (STC) schemes defined by Tarokh et al., 1998a and Alamouti, 1998a which introduce temporal and spatial correlation into the signals transmitted from different antennas without increasing the total transmitted power or the transmission bandwidth. There is in fact a diversity gain that results from multiple paths between the base station and user terminal, and a coding gain that results from how symbols are correlated across transmit antennas. Significant performance improvements are possible with only two antennas at the base station and one or two antennas at the user terminal, and with simple receiver structures. The second antenna at the user terminal can be used to further increase system capacity through interference suppression.

In only a few years, space-time codes have progressed from invention to adoption in the major wireless standards. For WCDMA where short spreading sequences are used, transmit diversity provided by space-time codes represents the difference between data rates of 100 kb/s and 384 kb/s. Our emphasis is on solutions that include channel estimation, joint decoding and equalization, and where the complexity of signal processing is practical. The new world of multiple transmit and receive antennas requires significant modification of techniques developed for single-transmit single-receive communication. Since receiver cost and complexity is an important consideration, our treatment of innovation in signal processing is grounded in systems with 1, 2 or 4 transmit antennas and 1 or 2 receive antennas. For example, the interference cancellation techniques presented in Section 4 enable transmission of 1 Mb/s over a 200 kHz GSM/EDGE channel using 4 transmit and 2 receive antennas. Hence, our limitation on numbers of antennas does not significantly dampen user expectations.

Initial STC research efforts focused on narrowband flat-fading channels Tarokh et al., 1998a; Naguib et al., 1998; Alamouti, 1998a. Successful implementation of STC over multi-user broadband frequency-selective channels requires the development of novel, practical, and high-performance signal processing algorithms for channel estimation, joint equalization/decoding, and interference suppression. This task is quite challenging due to the long delay spread of broadband channels which increases the number of channel parameters to be estimated and the number of trellis states in joint equalization/decoding, especially with multiple transmit antennas. This, in turn, places significant additional computational and power consumption loads on user terminals. On the other hand, development and implementation of such advanced algorithms for broadband wireless channels promises even more significant performance gains than those reported for narrowband channels Tarokh et al., 1998a; Naguib et al., 1998; Alamouti, 1998a due to availability of multipath (in addition to spatial) diversity gains that can be realized. By virtue of their design, space-time-coded signals enjoy rich algebraic structure that can (and should!) be exploited to develop near-optimum reduced-complexity modem signal processing algorithms.

The organization of this chapter is as follows. We start in Section 2 with background material where we set up the broadband wireless channel model assumed followed by a discussion of transmit diversity and the concept of diversity order. Section 3 describes STC design criteria and discusses representative examples with both the trellis and block structure. We also give some recent developments in space-time codes. Section 4 shows through concrete examples from signal processing, coding theory, and networking, how the STC algebraic structure can be exploited to enhance system performance and reduce implementation complexity. The chapter concludes in Section 5 with a summary and discussion of several future challenges.

2. Background

2.1 Broadband Wireless Channel Model

A typical outdoor wireless propagation environment is represented in Figure 1.1 where the mobile wireless terminal is communicating with a wireless access point (base-station). The signal transmitted from the mobile may reach the access point directly (line-of-sight) or through multiple reflections on local scatterers (buildings, mountains etc.). As a result, the received signal is affected by multiple random attenuations and delays. Moreover, the mobility of either the nodes or the scattering environment may cause these random fluctuations to vary with time. Furthermore, a shared wireless environment may cause undesirable interference to the transmitted signal. This combination of factors makes wireless a challenging communication environment. For a transmitted

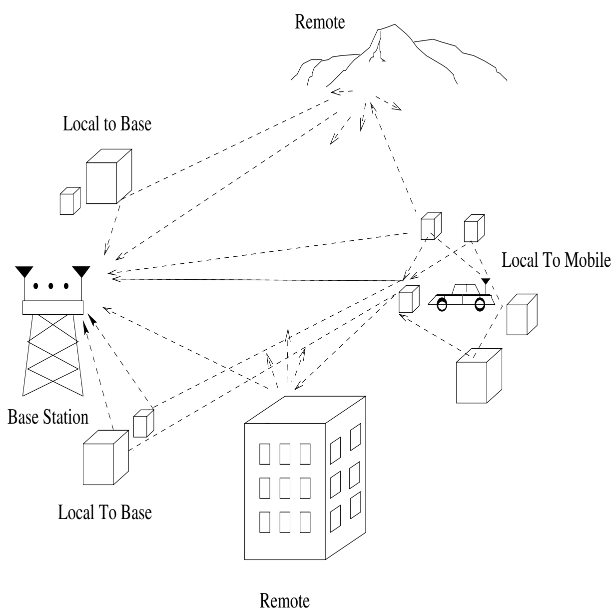

Figure 1.1. Radio Propagation Environment

signal $s(t)$, the continuous-time received signal $y_c(t)$ can be expressed as

$$y_c(t) = \int h_c(t;\tau)s(t-\tau)d\tau + z(t) \ , \tag{1.1}$$

where $h_c(t;\tau)$ is the response of the time-varying channel [1] if an impulse is sent at time $t-\tau$, and $z(t)$ is the additive Gaussian noise. To collect discrete-time sufficient statistics we need to sample (1.1) faster than the Nyquist rate. That is, we sample (1.1) at a rate larger than $2(W_I + W_s)$, where W_I is the input bandwidth and W_s is the bandwidth of the channel time-variation. In this article, we assume that this criterion is met and therefore we focus on the following discrete-time model

$$y(k) = y_c(kT_s) = \sum_{l=0}^{\nu} h(k;l)x(k-l) + z(k) \ , \tag{1.2}$$

where $y(k)$, $x(k)$, and $z(k)$ are the output, input, and noise samples at sampling instant k, respectively, and $h(k;l)$ represents the sampled time-varying channel impulse response (CIR) of finite memory ν. Any loss in modeling the channel as having a finite-duration impulse response can be made small by appropriately selecting ν.

Three key characteristics of broadband mobile wireless channels are time-selectivity, frequency-selectivity, and space-selectivity. Time-selectivity arises from mobility, frequency-selectivity arises from broadband transmission, and space-selectivity arises from the spatial interference patterns of the radio waves. The corresponding key parameters in the characterization of mobile broadband wireless channels are *coherence time*, *coherence bandwidth*, and *coherence distance*. The coherence time is the time duration over which each CIR tap can be assumed constant. It is approximately equal to the inverse of the Doppler frequency[2]. The channel is said to be *time-selective* if the symbol period is larger than

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[1]Including the effects of transmit/receive filters.

[2]The Doppler frequency is a measure of the frequency spread experienced by a pure sinusoid transmitted over the channel. It is equal to the ratio of the mobile speed to the carrier wavelength.

the channel coherence time. The coherence bandwidth is the frequency duration over which the channel frequency response can be assumed flat. It is approximately equal to the inverse of the channel delay spread[3]. The channel is said to be *frequency-selective* if the symbol period is smaller than the delay spread of the channel. Likewise, the coherence distance is the maximum spatial separation over which the channel response can be assumed constant. This can be related to the behaviour of arrival directions of the reflected radio waves and is characterized by the *angular spread* of the multiple paths Jakes, 1974; Rappaport, 1996. The channel is said to be space-selective between two antennas if their separation is larger than the coherence distance.

The channel memory causes interference among successive transmitted symbols that results in significant performance degradation unless corrective measures (known as equalization) are implemented. In this chapter, we shall use the terms *frequency-selective channel*, *broadband channel*, and *intersymbol interference (ISI) channel* interchangeably. The introduction of M_t transmit and M_r receive antennas leads to the following generalization of the basic channel model

$$\mathbf{y}(k) = \sum_{l=0}^{\nu} \mathbf{H}(k;l)\mathbf{x}(k-l) + \mathbf{z}(k) \, , \tag{1.3}$$

where the $M_r \times M_t$ complex matrix $\mathbf{H}(k;l)$ represents the l^{th} tap of the channel matrix response with $\mathbf{x} \in \mathbb{C}^{M_t}$ as the input and $\mathbf{y} \in \mathbb{C}^{M_r}$ as the output. The input vector may have independent entries to achieve high throughput (e.g. through spatial multiplexing) or correlated entries through coding or filtering to achieve high reliability (better distance properties, higher diversity, spectral shaping, or desirable spatial profile). Throughout this article, the input is assumed to be zero mean and satisfy an average power constraint i.e., $\mathbb{E}[||\mathbf{x}(k)||^2] \leq P$. The vector

[3]The channel delay spread is a measure of the time spread experienced by a pure impulse transmitted over the channel.

$\mathbf{z} \in \mathbb{C}^{M_r}$ models the effects of noise and interference[4]. It is assumed to be independent of the input and is modeled as a complex additive circularly-symmetric Gaussian vector with $\mathbf{z} \sim \mathbb{C}\mathcal{N}(0, \mathbf{R}_{zz})$, i.e., a complex Gaussian vector with mean $\mathbf{0}$ and covariance $\mathbf{R}_{zz}$. Finally, we modify the basic channel model to accommodate a block or frame of N consecutive symbols. Now, Equation (1.3) can be expressed in matrix notation as follows

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{z} \;, \tag{1.4}$$

where $\mathbf{y}, \mathbf{z} \in \mathbb{C}^{N.M_r}$, $\mathbf{x} \in \mathbb{C}^{M_t(N+\nu)}$, and $\mathbf{H} \in \mathbb{C}^{N.M_r \times M_t(N+\nu)}$. In each input block, we insert a guard sequence of length equal to the channel memory ν to eliminate inter-block interference (IBI). In practice, the most common choices for the guard sequence are the all-zeros sequence (also known as *zero stuffing*) and the *cyclic prefix* (CP). When the channel is known at the transmitter, it is possible to increase throughput by optimizing the choice of the guard sequence.

The channel model in (1.4) includes several popular special cases. First, the quasi-static channel model follows by assuming the channel time-invariant within the transmission block. In this case, using the cyclic prefix makes the channel matrix $\mathbf{H}$ *block-circulant*, hence diagonalizable using the Fast Fourier Transform (FFT). Second, the flat-fading channel model follows by setting $\nu = 0$ which renders the channel matrix $\mathbf{H}$ a *block diagonal* matrix. Third, the channel model for single-antenna transmission, reception, or both follows directly by setting M_t, M_r, or both equal to 1, respectively.

2.2 Transmit Diversity

Transmit diversity is more challenging to provision and realize than receive diversity because it involves the design of multiple correlated signals from a single information signal without utilizing CSI (typically not

[4] Including co-channel interference, adjacent channel interference, and multi-user interference.

available accurately at the transmitter). Furthermore, transmit diversity must be coupled with effective receiver signal processing techniques that can extract the desired information signal from the distorted and noisy received signal. Transmit diversity is more practical than receive diversity for enhancing the downlink (which is the bottleneck in broadband asymmetric applications such as Internet browsing and downloading) to preserve the small size and low power consumption features of the user terminal. A common attribute of transmit and receive diversity is that both experience "diminishing returns" (i.e. diminishing SNR gains at a given error of probability) as the number of antennas increases Jakes, 1974. This makes them effective, from a performance-complexity trade-off point of view, for small numbers of antennas (typically less than four). This is in contrast with spatial multiplexing gains where the rate continues to increase linearly with the number of antennas (assumed equal at both ends).

There are two main classes of multiple-antennas transmitter techniques : closed-loop and open-loop. The former uses a feedback channel to send CSI acquired at the receiver back to the transmitter to be used in signal design while the latter does not require CSI. Assuming availability of ideal (i.e. error free and instantaneous) CSI at the transmitter, closed-loop techniques have an SNR advantage of $10\log_{10}(M_t)$ dB over open-loop techniques due to the "array gain" factor Alamouti, 1998a. However, several practical factors degrade the performance of closed-loop techniques including channel estimation errors at the receiver, errors in feedback link (due to noise, interference, and quantization effects), and feedback delay which causes a mismatch between available and actual CSI. All of these factors combined with the extra bandwidth and system complexity resources needed for the feedback link make open-loop techniques more attractive as a robust means for improving downlink performance for high-mobility applications while closed-loop techniques (such as beamforming) become attractive under low-mobility conditions.

Our focus in this chapter will be exclusively on open-loop spatial transmit diversity techniques due to their applicability to both scenarios[5]. Beamforming techniques are discussed extensively in several tutorial papers such as Godara, 1997a; Godara, 1997b.

The simplest example of open-loop spatial transmit diversity techniques is *delay diversity* Uddenfeldt and Raith, 1992; Wittneben, 1993 where the signal transmitted at sampling instant k from the i^{th} antenna is $x_i(k) = x(k-l_i)$ for $2 \leq i \leq M_t$ and $x_1(k) = x(k)$ where l_i denotes the time delay (in symbol periods) on the i^{th} transmit antenna. Assuming a single receive antenna, the D-transform[6] of the received signal is given by

$$y(D) = x(D)(h_1(D) + \sum_{i=2}^{M_t} D^{l_i} h_i(D)) + z(D) \ . \tag{1.5}$$

It is clear from (1.5) that delay diversity transforms spatial diversity into multipath diversity that can be realized through equalization Seshadri and Winters, 1993. For flat-fading channels, we can set $l_i = (i-1)$ and achieve full (i.e. order-M_t) spatial diversity using an ML equalizer with $(2^b)^{M_t-1}$ states where 2^b is the input alphabet size. However, for frequency-selective channels, a delay of at least $l_i = (i-1)(\nu+1)$ is needed to ensure that coefficients from the various spatial FIR channels do not interfere with each other causing a diversity loss. This, in turn, increases equalizer complexity to $(2^b)^{(M_t-1)(\nu+1)}$ states which is prohibitive even for moderate b, M_t, and ν. In Section 3, we describe another family of open-loop spatial transmit diversity techniques known as space-time block codes that achieve full spatial diversity with practical complexity even for frequency-selective channels with long delay spread.

[5]It is also possible to combine closed-loop and open-loop techniques as shown recently in Soni et al., 2002.

[6]The D-transform of a discrete-time sequence $\{x(k)\}_{k=0}^{N-1}$ is defined as $x(D) = \sum_{k=0}^{N-1} x(k)D^k$. It is derived from the Z-transform by replacing the unit delay Z^{-1} by D.

2.3 Diversity Order

Error probability is particularly important as a performance criterion when we are coding over a small number of blocks (low-delay) where the Shannon capacity is zero Ozarow et al., 1994 and, therefore, we need to design for low error probability. By characterizing the error probability, we can also formulate design criteria for space-time codes in Section 3

Since we are allowed to transmit a coded sequence, we are interested in the probability that an erroneous codeword $\mathbf{e}$ is mistaken for the transmitted codeword $\mathbf{x}$. This is called the *pairwise error probability* (PEP) and is then used to bound the error probability. This is analyzed under the condition that the receiver has perfect channel state information. However, a similar analysis can be performed when the receiver does not know the channel state information but has statistical knowledge of the channel.

For simplicity, we shall first present the result for a flat-fading Rayleigh channel (where $\nu = 0$). In the case when the receiver has perfect channel state information, we can bound the PEP between $\mathbf{x}$ and $\mathbf{e}$ (denoted by $P(\mathbf{x} \to \mathbf{e})$) as follows Tarokh et al., 1998a; Guey et al., 1999

$$P(\mathbf{x} \to \mathbf{e}) \leq \left[\frac{1}{\prod_{n=1}^{M_t}(1 + \frac{E_s}{4N_0}\lambda_n)} \right]^{M_r}, \tag{1.6}$$

where λ_n are the eigenvalues of the matrix $\mathbf{A}(\mathbf{x},\mathbf{e}) = \mathbf{B}^*(\mathbf{x},\mathbf{e})\mathbf{B}(\mathbf{x},\mathbf{e})$ and

$$\mathbf{B}(\mathbf{x},\mathbf{e}) = \begin{pmatrix} \mathbf{x}_1(1) - \mathbf{e}_1(1) & \dots & \mathbf{x}_{M_t}(0) - \mathbf{e}_{M_t}(0) \\ \vdots & \vdots & \vdots \\ \mathbf{x}_1(N-1) - \mathbf{e}_1(N-1) & \dots & \mathbf{x}_{M_t}(N-1) - \mathbf{e}_{M_t}(N-1) \end{pmatrix}. \tag{1.7}$$

If q denotes the rank of $\mathbf{A}(\mathbf{x}, \mathbf{e})$, (*i.e.*, the number of non-zero eigenvalues) then we can rewrite (1.6) as

$$P(\mathbf{x} \to \mathbf{e}) \leq \left[\prod_{n=1}^{q} \lambda_n\right]^{-M_r} \left(\frac{E_s}{4N_0}\right)^{-qM_r}. \tag{1.8}$$

Thus, we define the notion of diversity order as follows.

Definition 1.1 *A scheme which has an average error probability $\bar{P}_e(SNR)$ as a function of SNR that behaves as*

$$\lim_{SNR \to \infty} \frac{\log(\bar{P}_e(SNR))}{\log(SNR)} = -d \tag{1.9}$$

is said to have a diversity order of d.

In words, a scheme with diversity order d has an error probability at high SNR behaving as $\bar{P}_e(SNR) \approx SNR^{-d}$. Given this definition, we can see that the diversity order in (1.8) is at most qM_r. Moreover, in (1.8) we obtain an additional coding gain of $(\prod_{n=1}^{q} \lambda_n)^{1/q}$.

Note that in order to obtain the average error probability, one can calculate a naive union bound using the pairwise error probability given in (1.8). However, this bound may not be tight and a more careful upper bound for the error probability can be derived Zheng and Tse, 2003; Siwamogsatham et al., 2002. However, if we ensure that *every* pair of codewords satisfies the diversity order in (1.8), then clearly the average error probability satisfies it as well. This is true when the transmission rate is held constant with respect to SNR. Therefore, code design for diversity order through pairwise error probability is a sufficient condition, although more detailed criteria can be derived based on a more accurate expression for average error probability.

The error probability analysis can easily be extended to the case where we have quasi-static ISI channels with channel taps modeled as i.i.d. zero-mean complex Gaussian random variables (see for example Zhou and Giannakis, 2001 and references therein). In this case, the PEP can be written as

$$P(\mathbf{x} \to \mathbf{e}) \le \left[\frac{1}{\prod_{n=1}^{M_t \nu} (1 + \frac{E_s}{4N_0} \tilde{\lambda}_n)} \right]^{M_r}, \tag{1.10}$$

where the eigenvalues $\tilde{\lambda}_n$ are those of $\tilde{\mathbf{A}}(\mathbf{x}, \mathbf{e}) = \tilde{\mathbf{B}}^*(\mathbf{x}, \mathbf{e})\tilde{\mathbf{B}}(\mathbf{x}, \mathbf{e})$,

$$\tilde{\mathbf{B}}(\mathbf{x}, \mathbf{e}) = \begin{pmatrix} \tilde{\mathbf{x}}^T(0) - \tilde{\mathbf{e}}^T(0) \\ \vdots \\ \tilde{\mathbf{x}}^T(N-1) - \tilde{\mathbf{e}}^T(N-1) \end{pmatrix}, \tag{1.11}$$

and

$$\tilde{\mathbf{x}}(k) = [\mathbf{x}^T(k), \ldots, \mathbf{x}^T(k - \nu)]^T. \tag{1.12}$$

Since $\tilde{\mathbf{A}}(\mathbf{x}, \mathbf{e})$ is a square matrix of size $M_t \nu$, clearly the maximal diversity order achievable for quasi-static ISI channels is $M_r M_t \nu$.

Finally, if we have a time-varying ISI channel, we can generalize (1.10) to

$$P(\mathbf{x} \to \mathbf{e}) \le \left[\frac{1}{|\mathbf{I}_{M_r N M_t \nu} + \frac{E_s}{4N_0} \mathbf{F}(\mathbf{R}_h \otimes \mathbf{I}_{M_r M_t \nu})|} \right], \tag{1.13}$$

where $\otimes$ denotes a Kronecker product, $\mathbf{R}_h$ is the $N \times N$ correlation matrix of the channel tap process, and $\mathbf{F} = diag\{\mathbf{C}(0), \ldots, \mathbf{C}(N-1)\}$ with

$$\mathbf{C}(k) = \left[\tilde{\mathbf{x}}^T(k) - \tilde{\mathbf{e}}^T(k) \right] \otimes \mathbf{I}_{M_r}. \tag{1.14}$$

Again, it is clear that the maximal diversity attainable is $M_r M_t \nu N$, but for a given channel tap process, N is replaced by the number of dominant eigenvalues N_{dom} of the fading correlation matrix. This parameter is related to the Doppler spread of the channel and the block duration.

2.4 Rate-diversity trade-off

A natural question that arises is how many codewords can we have which allow us to attain a certain diversity order. For a flat Rayleigh fading channel, this has been examined in Tarokh et al., 1998b; Lu and Kumar, 2003 and the following result was obtained[7].

Theorem 1.2 *If we use a constellation of size* 2^b *and the diversity order of the system is* qM_r*, then the rate* R *that can be achieved is bounded as*

$$R \leq (M_t - q + 1) \log_2 |\mathcal{S}| \tag{1.15}$$

in bits per transmission.

One consequence of this result is that for maximum $(M_t M_r)$ diversity order we can transmit at most b bits/sec/Hz.

An alternate viewpoint in terms of the rate-diversity trade-off has been explored in Zheng and Tse, 2003 from a Shannon-theoretic point of view. Here the authors are interested in the multiplexing rate of a transmission scheme.

Definition 1.3 *A coding scheme which has a transmission rate of* $R(SNR)$ *as a function of* SNR *is said to have a multiplexing rate* r *if*

$$\lim_{SNR \to \infty} \frac{R(SNR)}{\log(SNR)} = r. \tag{1.16}$$

Therefore, the system has a rate of $r \log(SNR)$ at high SNR. One way to contrast it with the statement in Theorem 1.2, is that the constellation size is also allowed to become larger with SNR. However, note that the naive union bound of the pairwise error probability (1.6) has to be used with care if the constellation size is also increasing with SNR. There is a trade-off between the achievable diversity and the multiplexing rate, and $d^{opt}(r)$ is defined as the supremum of the diversity gain achievable by *any* scheme with multiplexing rate r. The main result in Zheng and Tse, 2003 states that

[7] A constellation size refers to the alphabet size of each transmitted symbol. For example, a QPSK modulated transmission has constellation size of 4.

THEOREM 1.4 *For $N > M_t + M_r - 1$, and $K = \min(M_t, M_r)$, the optimal trade-off curve $d^{opt}(r)$ is given by the piece-wise linear function connecting points in $(k, d^{opt}(k)), k = 0, \ldots, K$ where*

$$d^{opt}(k) = (M_r - k)(M_t - k). \tag{1.17}$$

■

The interesting interpretation of this result is that one can get large rates which grow with SNR if we reduce the diversity order from the maximum achievable. This diversity-multiplexing tradeoff implies that a high multiplexing rate comes at the price of decreased diversity gain and is a manifestation of a corresponding tradeoff between error probability and rate.

A different question was proposed in Diggavi et al., 2004a; Diggavi et al., 2003a, where it was asked whether there exists a strategy that combines high-rate communications with high-reliability (diversity). Clearly the overall code will still be governed by the rate-diversity trade-off, but the idea is to ensure the reliability (diversity) of at least part of the total information. This allows a form of communication where the high-rate code opportunistically takes advantage of good channel realizations whereas the embedded high-diversity code ensures that at least part of the information is received reliably. In this case, the interest was not in a single pair of multiplexing rate and diversity order (r, d), but in a tuple (r_a, d_a, r_b, d_b) where rate r_a and diversity order d_a was ensured for part of the information with rate-diversity pair (r_b, d_b) guaranteed for the other part. A class of space-time codes with such desired characteristics will be discussed in Section 3.5.

From an information-theoretic point of view Diggavi and Tse, 2004; Diggavi and Tse, 2005 focused on the case when there is one degree of freedom (*i.e.,* $\min(M_t, M_r) = 1$). In that case if we consider $d_a \geq d_b$ without loss of generality, the following result was established in Diggavi and Tse, 2004; Diggavi and Tse, 2005.

Theorem 1.5 *When* $\min(M_t, M_r) = 1$*, then the diversity-multiplexing trade-off curve is successively refinable, i.e., for any multiplexing rates* r_a *and* r_b *such that* $r_a + r_b \leq 1$*, the diversity orders* $d_a \geq d_b$*,*

$$d_a = d^{opt}(r_a), \;\; d_b = d^{opt}(r_a + r_b) \tag{1.18}$$

are achievable, where $d^{opt}(r)$ *is the optimal diversity order given in Theorem 1.4.*

■

Since the overall code has to still be governed by the rate-diversity trade-off given in Theorem 1.4, it is clear that the trivial outer bound to the problem is that $d_a \leq d^{opt}(r_a)$ and $d_b \leq d^{opt}(r_a + r_b)$. Hence Theorem 1.5 shows that the best possible performance can be achieved. This means that for $\min(M_t, M_r) = 1$, we can design ideal *opportunistic* codes. This new direction of enquiry is being currently explored (see Diggavi and Tse, 2005; Diggavi et al., 2005).

3. Space-Time Coding Principles

Space-time coding has received considerable attention in academic and industrial circles Al-Dhahir et al., 2002b; Al-Dhahir et al., 2002a due to its many advantages. First, it improves the downlink performance without the need for multiple receive antennas at the terminals. For example, for WCDMA, STC techniques where shown in Parkvall et al., 2000 to result in substantial capacity gains due to the resulting "smoother" fading which, in turn, makes power control more effective and reduces the transmitted power. Second, it can be elegantly combined with channel coding, as shown in Tarokh et al., 1998a, realizing a coding gain in addition to the spatial diversity gain. Third, it does not require CSI at the transmitter, i.e. operates in open-loop mode, thus eliminating the need for an expensive and, in case of rapid channel fading, unreliable reverse link. Finally, they have been shown to be robust against non-ideal operating conditions such as antenna correlation, channel estimation errors, and Doppler effects Tarokh et al., 1999b; Naguib et al., 1998. There has been extensive work on the design of space-time codes since its introduction in Tarokh et al., 1998a. The combination of the turbo principle Berrou and Glavieux, 1996; Benedetto and Montorsi, 1996 with space-time codes has been explored (see for example Bauch and Al-Dhahir, 2002; Liu et al., 2001b among several other references). In addition, the application of linear density parity check (LDPC) codes Gallager, 1963 to space-time coding has been explored (see for example Lu et al., 2002 and references therein). We focus our discussion on the basic principles of space-time codes and describe next two main flavors: trellis and block codes.

3.1 Space-Time Code Design Criteria

In order to design practical codes that achieve a performance target we need to glean insights from the analysis to arrive at design criteria. For example, in the flat-fading case of (1.8) we can state the following rank and determinant design criteria Tarokh et al., 1998b; Guey et al., 1999.

- *Rank criterion:* In order to achieve maximum diversity $M_t M_r$, the matrix $\mathbf{B}(\mathbf{x}, \mathbf{e})$ from (1.7) has to be full rank for any codewords $\mathbf{x}, \mathbf{e}$. If the minimum rank of $\mathbf{B}(\mathbf{x}, \mathbf{e})$ over all pairs of distinct codewords is q, then a diversity order of qM_r is achieved.

- *Determinant criterion:* For a given diversity order target of q, maximize $(\prod_{n=1}^{q} \lambda_n)^{1/q}$ over all pairs of distinct codewords.

A similar set of design criteria can be stated for the quasi-static ISI fading channel using the PEP given in (1.10) and the corresponding error matrix given in (1.11). Therefore, if we need to construct codes satisfying these design criteria, we can guarantee performance in terms of diversity order. The main problem in practice is to construct such codes which do not have large decoding complexity. This sets up a familiar tension on the design in terms of satisfying the performance requirements and having low-complexity decoding.

If coherent detection is difficult or too costly, one can employ non-coherent detection for the multiple-antenna channel Hochwald and Marzetta, 1999; Zheng and Tse, 2002b. Though it is demonstrated in Zheng and Tse, 2002b that a training-based technique achieves the same capacity-SNR slope as the optimal, there might be a situation where inexpensive receivers are needed because channel estimation cannot be accommodated. In such a case, differential techniques which satisfy the diversity order might be desirable. There has been significant work on differential transmission with non-coherent detection (see for example Hughes, 2000; Hochwald and Sweldens, 2000 and references therein) and this is a topic we discuss briefly in Section 4.1.

The rank and determinant design criteria given above are suitable for transmission when we have a fixed input alphabet. As mentioned in Section 2.4, the rate-diversity trade-off can also be explored in the context of the multiplexing rate (see definition 1.3). Therefore, a natural question to ask is the code-design criteria in this context. For the diversity-multiplexing guarantees, it is not clear that the rank and determinant criterion is the correct one to use. In fact, in El-Gamal et al., 2004, it is shown that for designing codes with the multiplexing rate in mind, the determinant criterion is not necessary for *specific* fading distributions. However, it has been shown that the determinant criterion again arises as a sufficient condition for designing codes for the diversity-multiplexing rate trade-off for specific constructions (see Yao and Wornell, 2003; Elia et al., and references therein). For these constructions, it is shown that the determinant of the code-word difference matrix plays a crucial role in the diversity-multiplexing optimality of the codes.

Another multiplexing rate context in which the code-word difference matrix plays a crucial role in the space-time code design is in the design of *approximately universal* codes Tavildar and Viswanath, 2006. Traditionally space-time codes are designed for a *particular* distribution of the channel. Universal codes codes are designed to give an error probability which decays *exponentially* in SNR for all channels that are not in outage. Therefore, this gives a robust design rule which gives performance guarantees over the worst case channel, rather than averaging over the channel statistics. For the multiple-transmit single-receive (MISO) channel, the code design is related to maximizing the *smallest* singular value of the code-word difference matrix. This corresponds to a worst-case channel that aligns itself with the weakest direction of the code-word difference matrix. This is in contrast to the *average* case, where we are interested in maximizing the product of the singular values (*i.e.,* the determinant). In fact for MIMO channels, in certain cases, the maximizing the determinant of the code-word difference matrix again arises as the universal code design criterion Tavildar and Viswanath, 2006.

3.2 Space-Time Trellis Codes (STTC)

The space-time trellis encoder maps the information bit stream into M_t streams of symbols (each belonging in a size-2^b signal constellation) that are transmitted simultaneously[8]. STTC design criteria are based on minimizing the PEP bound in Section 2.3.

As an example, we consider the 8-state 8-PSK STTC for two transmit antennas introduced in Tarokh et al., 1998a; the trellis description is given in Figure 1.2, where the edge label c_1c_2 means that symbol c_1 is transmitted from the first antenna and symbol c_2 from the second antenna. The different symbol pairs in a given row label the transitions out of a given state, in order, from top to bottom. An equivalent and convenient (for reasons to become clear shortly) implementation of the 8-state 8-PSK STTC encoder is depicted in Figure 1.3. This equivalent implementation clearly shows that the 8-state 8-PSK STTC is identical

[8]The total transmitted power is divided equally among the M_t transmit antennas.

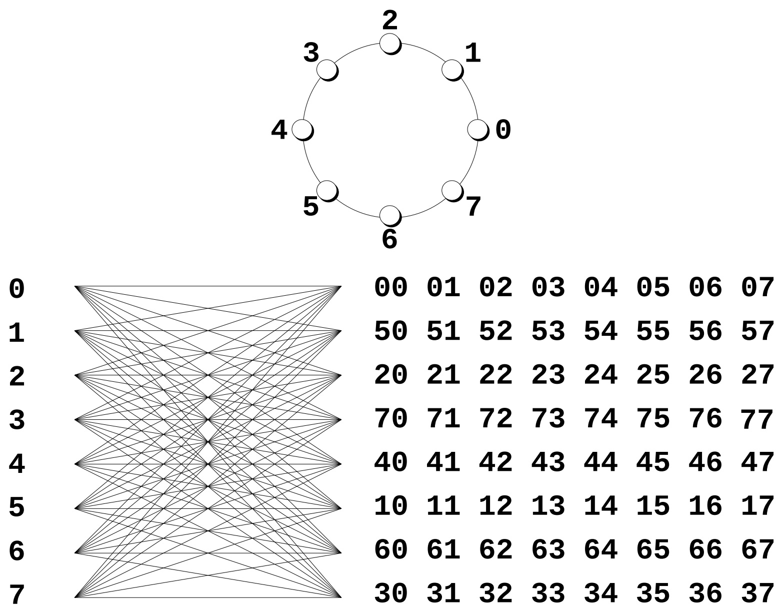


Figure 1.2. 8-State 8-PSK Space-Time Trellis Code with Two Transmit Antennas and a Spectral Efficiency of 3 bits/sec/Hz

to classical delay diversity transmission Seshadri and Winters, 1993 *except* that the delayed symbol from the second antenna is multiplied by -1 if it is an odd symbol, i.e. $\in \{1, 3, 5, 7\}$. This slight modification results in additional coding gain over a flat-fading channel. We emphasize that this STTC does not achieve the maximum possible diversity gains (spatial and multipath) on frequency-selective channels; however, its performance is near optimum for practical ranges of SNR on wireless links Fragouli et al., 2002[9]. Furthermore, when implementing the 8-state 8-PSK STTC described above on a frequency-selective channel, its structure can be exploited to reduce the complexity of joint equalization and decoding. This is achieved by embedding the space-time encoder in Figure 1.3 in the two channels $h_1(D)$ and $h_2(D)$ resulting

[9]For examples of STTC designs for frequency-selective channels see e.g. Liu et al., 2001a

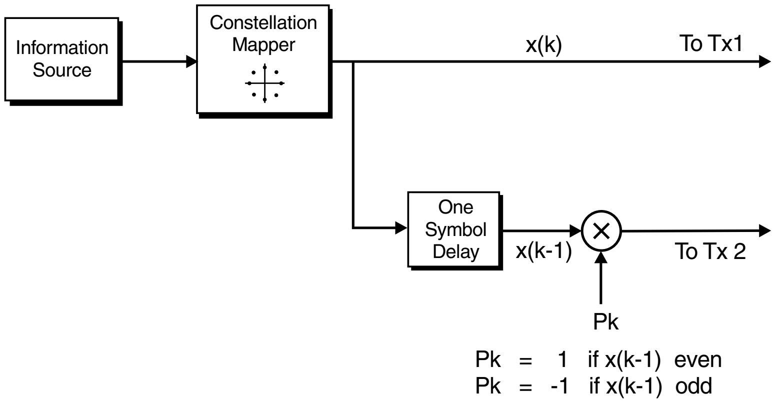


Figure 1.3. Equivalent Encoder Model for 8-State 8-PSK Space-Time Trellis Code with Two Transmit Antennas

in an equivalent single-input single-output (SISO) data-dependent CIR with memory $(\nu + 1)$ whose D-transform is given by

$$h_{eqv}^{STTC}(k, D) \quad = \quad h_1(D) + p_k D h_2(D) \; , \tag{1.19}$$

where $p_k = \pm 1$ is data-dependent. Therefore, trellis-based joint space-time equalization and decoding with $8^{\nu+1}$ states can be performed on this equivalent channel. Without exploiting the STTC structure, trellis equalization requires $8^{2\nu}$ states and STTC decoding requires 8 states.

The discussion in this section just illustrates one STTC example. Several other full-rate full-diversity STTC's for different signal constellations and different numbers of antennas were presented in Tarokh et al., 1998a.

3.3 Space-Time Block Codes (STBC)

The decoding complexity of STTC (measured by the number of trellis states at the decoder) increases *exponentially* as a function of the diversity level and transmission rate Tarokh et al., 1998a. In address-

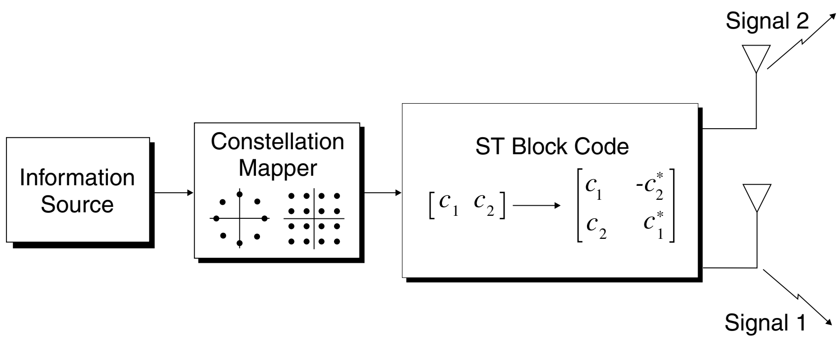


Figure 1.4. Spatial Transmit Diversity with Alamouti's Space-Time Block Code

ing the issue of decoding complexity, Alamouti, 1998a discovered an ingenious space-time block coding scheme for transmission with two antennas. According to this scheme (see also Appendix A), input symbols are grouped in pairs where symbols x_k and x_{k+1} are transmitted at time k from the first and second antennas, respectively. Then, at time $k+1$, symbol $-x_{k+1}^*$ is transmitted from the first antenna and symbol x_k^* is transmitted from the second antenna, where $*$ denotes the complex conjugate transpose (c.f. Figure 1.4). This imposes an orthogonal spatio-temporal structure on the transmitted symbols. Alamouti's STBC has been adopted in several wireless standards such as WCDMA T.I, and CDMA2000 TIA, 1998 due to the following attractive features. First, It achieves full diversity at full transmission rate for any (real or complex) signal constellation. Second, it does not require CSI at the transmitter (i.e. open loop). Third, maximum likelihood decoding involves only *linear* processing at the receiver (due to the orthogonal code structure).

The Alamouti STBC has been extended to the case of more than two transmit antennas Tarokh et al., 1999a using the theory of orthogonal designs. There it was shown that, in general, full-rate orthogonal designs exist for all real constellations for $2, 4, 8$ transmit antennas only while for all complex constellations they exist only for 2 transmit antennas (the Alamouti scheme). However, for particular constellations, it might be possible to construct full-rate orthogonal designs for larger numbers of transmit antennas. Moreover, if a rate loss is acceptable, then orthogonal designs exist for an arbitrary number of transmit antennas Tarokh et al., 1999a.

The advantage of orthogonal design is the simplicity of the decoder. However, using a sphere decoder, space-time codes that are not orthogonal, but are *linear* over the complext field can also be decoded efficiently. A class of space-time codes known as *Linear Dispersion Codes* (LDC) was introduced in Hassibi and Hochwald, 2002 where the orthogonality constraint is relaxed to achieve higher rate while still enjoying (expected) polynomial decoding complexity for a wide SNR range by using the sphere decoder. This comes at the expense of signal constellation expansion and not guaranteeing maximum diversity gains (as in orthogonal designs). With M_t transmit antennas and a channel coherence time of T, the $T \times M_t$ transmitted signal space–time matrix $\mathbf{X}$ in LDC schemes has the form

$$\mathbf{X} = \sum_{q=1}^{Q} \alpha_q \mathbf{A}_q + j\beta_q \mathbf{B}_q \ , \tag{1.20}$$

where the real scalars α_q and β_q are related to the Q information symbols x_q (that belong to a size-2^b complex signal constellation) by $x_q = \alpha_q + j\beta_q$ for $q = 1, 2, \cdots Q$. This LDC has a rate of $\frac{Q}{T} \log_2 M$. Several LDC designs were presented in Hassibi and Hochwald, 2002 based on a judicious choice of the parameters T,Q and the so–called dispersion matrices $\mathbf{A}_q$ and $\mathbf{B}_q$ to maximize the mutual information between the transmitted and received signals.

An alternate way to attain diversity is to build in the diversity into the modulation through constellation rotations. This basic idea was proposed in Boulle and Belfiore, 1992; Kerpez, 1993 and developed for higher dimensional lattices in Boutros and Viterbo, 1998 and references therein. Therefore, one can construct modulation schemes with built-in diversity, with the caveat that the constellation size is actually increasing. The point to note here is that the Theorem 1.2 refers to rate versus diversity tradeoff for a given constellation size. Therefore, in order to consider the efficiency of coding schemes based on the rotated constellations, one needs to take into account the expansion in the constellation size. As an alternative to alphabet constraints, other constellation constraints have been studied in order to produce codes with maximal rate as well as maximal diversity order (see El-Gamal and Damen, 2003 and references therein). Therefore, constellation rotations without alphabet constraints can yield the maximal performance of *both* rate (in terms of information constellation size) and diversity order. Note that in these cases there is a difference between the *information* constellation size and the constellation size of the transmitted codeword, which could be much larger.

Therefore, in this sense, the rotated codes are actually more suitable in the context of *diversity-multiplexing* trade-off discussed in Section 2.4, where there are no transmit alphabet constraints. In fact, using such rotation based codes, several diversity-multiplexing rate optimal codes have been constructed (see Yao and Wornell, 2003; Elia et al., ; Tavildar and Viswanath, 2006 and references therein).

Recently, STBCs have been extended to the frequency-selective channel case by implementing the Alamouti orthogonal signaling scheme at a *block* level instead of *symbol* level. Depending on whether the implementation is done in the time or frequency domain, three STBC structures for frequency-selective channels have been proposed : Time-Reversal (TR)-STBC Lindskog and Paulraj, 2000, OFDM-STBC Liu et al., 1999, and Frequency-Domain-Equalized (FDE)-STBC Al-Dhahir, 2001. As an illustration, we present next the space-time encoding scheme for FDE-STBC. Denote the n^{th} symbol of the k^{th} transmitted block from antenna i by $x_i^{(k)}$. At times $k = 0, 2, 4, \ldots$ pairs of length-N blocks $\mathbf{x}_1^{(k)}(n)$ and $\mathbf{x}_2^{(k)}(n)$ (for $0 \leq n \leq N-1$) are generated by the mobile user. Inspired by Alamouti's STBC, we encode the information symbols as follows Al-

Dhahir, 2001

$$\begin{aligned} &\mathbf{x}_1^{(k+1)}(n) = -\mathbf{x}_2^{*(k)}((-n)_N) \text{ and } \mathbf{x}_x^{(k+1)}(n) = \mathbf{x}_1^{*(k)}((-n)_N) \\ &\quad \text{for } n = 0, 1, \ldots, N-1 \text{ and } k = 0, 2, 4, \ldots \end{aligned} \tag{1.21}$$

where $(\cdot)_N$ denotes the modulo-N operation. In addition, a cyclic prefix of length ν (the maximum order of the FIR wireless channel) is added to each transmitted block to eliminate IBI and make all channel matrices *circulant*. We refer the reader to Al-Dhahir, 2002 for a detailed description and comparison of these schemes. The main point we would like to stress here is that these three STBC schemes can realize both spatial and multipath diversity gains at practical complexity levels. For channels with long delay spread, the frequency-domain implementation using the FFT either in a single-carrier or multicarrier fashion becomes more advantageous from a complexity point of view.

3.4 A New Non-Linear Maximum-Diversity Quaternionic Code

In this section, we show how the STC algebraic structure inspires new code designs with desirable rate-diversity characteristics and low decoding complexity.

The only full-rate complex orthogonal design is the 2×2 Alamouti code Alamouti, 1998b, and as the number of transmit antennas increases, the available rate becomes unattractive. For example, for 4 transmit antennas, orthogonal STBC designs with rates $\frac{1}{2}$ and $\frac{3}{4}$ were presented in Tarokh et al., 1999a. This rate limitation of orthogonal designs caused a recent shift of research focus to non-orthogonal code design. These include a quasi-orthogonal design Jafarkhani, 2001 for 4 transmit antennas that has rate 1 but achieves only second-order diversity. Full diversity can be achieved by including signal rotations which expand the constellation. Another approach is the design of non-orthogonal but linear codes Damen et al., 2002 for which decoding is efficient albeit not linear in complexity. In this project, we revisit the problem of designing orthogonal STBC for 4 TX. Another reason for our interest in orthogonal designs is that they limit the SNR loss incurred by differential decoding to its minimum of 3 dB from coherent decoding. The proposed code in this proposal is constructed by means of 2×2 arrays over the quaternions, thus resulting in a 4×4 array over the complex field. The proposed code is rate-1, full-diversity (for any M-PSK constellation), orthogonal over the complex field, but is *not* linear. For QPSK, the code does not suffer constellation expansion and enjoys a simple maximum likelihood decoding algorithm. Consider the 4×4 space-time block code

$$\mathbf{X} = \begin{bmatrix} p & q \\ -q^* & \frac{q^* p^* q}{\|q\|^2} \end{bmatrix} \tag{1.22}$$

where each block entry is a quaternion. There is an isomorphism between quaternions $q = q_0 + iq_1 + jq_2 + kq_3$ and 2×2 complex matrices as follows

$$q \leftrightarrow \begin{bmatrix} q^c(0) & q^c(1) \\ -q^{*c}(1) & q^{*c}(0) \end{bmatrix} = \mathbf{Q}, \tag{1.23}$$

where $q^c(0) = q_0 + iq_1$, $q^c(1) = q_2 + iq_3$. Therefore, we may replace the quaternions p and q by the corresponding 2×2 complex matrices to obtain a 4×4 STBC with complex entries. There is a classical correspondence between unit quaternions and rotations in $\mathbf{R}^3$ given by $q \longrightarrow \mathbf{T}_q \ : \ p \longrightarrow q^* p q$ (details of the transformation $\mathbf{T}_q$ are given in Calderbank et al., 2005). For QPSK, maximum likelihood (ML) decoding requires a size-256 search. Through linear combining operations and appropriate application of the transformation $\mathbf{T}_q$, we showed how to exploit the quaternionic structure of this code to reduce the complexity of ML decoding to a size-16 search without loss of optimality.

In Figure 1.5, the significant performance gains achieved by the code in (1.22) in the IEEE 802.16 WiMAX environment Vaughan-Nichols, 2004 as compared to single-antenna transmission/reception translate to a 1.5 and 2.6 fold increase in the cell coverage area at 10^{-3} bit error rate when used with 1 and 2 receive antenna(s), respectively. We also

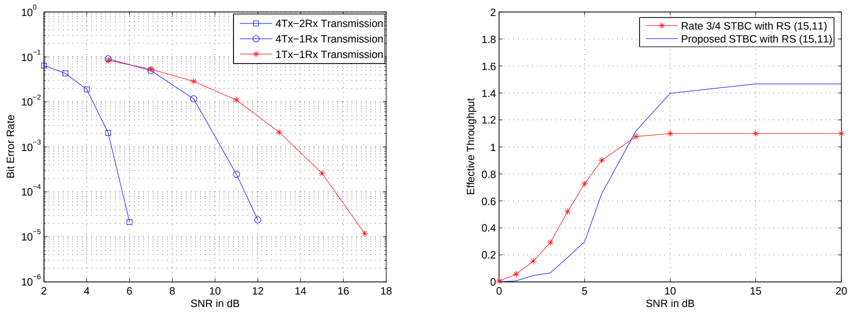


Figure 1.5. Left Figure Compares Quaternionic Code with Single-Antenna Case for WiMAX. Right Figure Compares Effective Throughput of Quaternionic Code with Octonion in a Quasi-Static Channel

compare in Figure 1.5 the effective throughput of our proposed quaternionic code with the rate-$\frac{3}{4}$ full-diversity Octonion code given in (A.1) assuming QPSK modulation and an outer RS$(15, 11)$ code for both. We observe that our proposed code achieves a throughput level of 1.46 bits per channel use whereas the achievable throughput for the Octonion code is 1.1 bits per channel use (33% increase).

3.5 Diversity-Embedded Space-Time Codes

The tradeoff between rate and diversity was explored within the framework of fixed alphabets by Tarokh, Seshadri, and Calderbank, Tarokh et al., 1998b and by Zheng and Tse Zheng and Tse, 2003 within an information-theoretic framework. Common to both is the observation that to achieve a high transmission rate, one must sacrifice diversity and vice-versa. As a consequence a large body of literature has mainly emphasized the design of codes that achieve a certain level of diversity (typically maximal diversity order), and a corresponding rate associated with it, *i.e.,* a particular point on this rate-diversity trade-off (see El-Gamal and Hammons, 2001,Lu and Kumar, 2005b and the references there).

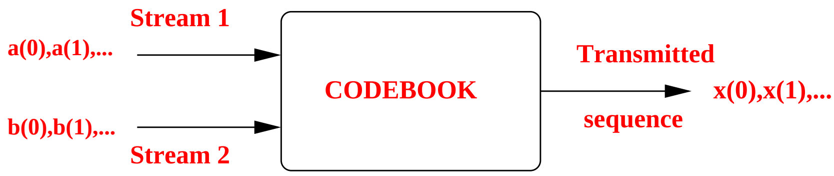


Figure 1.6. Embedded codebook

As explained at the end of Section 2.4 , a different point of view was proposed in Diggavi et al., 2003a; Diggavi et al., 2004a where the code was designed to achieve high rate but has embedded within it a higher-diversity (lower-rate) code (see Figure 1.6). Moreover, in this work it was argued that diversity can be viewed as a systems resource that can be allocated judiciously to achieve a desirable rate-diversity trade-off in wireless communications. In particular it was argued that if one designs the overall system for a fixed rate-diversity operating point, we might be over-provisioning a resource which could be flexibly allocated to different applications. For example, real-time applications need lower delay and therefore higher reliability (diversity) as compared to non-real time applications. By giving flexibility in the diversity allocation, one can simultaneously accommodate multiple applications with disparate rate-diversity requirements Diggavi et al., 2004a.

Let $\mathcal{A}$ denote the message set from the first information stream and $\mathcal{B}$ denote that from the second information stream. The rates for the two message sets are, respectively, $R(\mathcal{A})$ and $R(\mathcal{B})$. The decoder jointly decodes the two message sets with average error probabilities, $\bar{P}_e(\mathcal{A})$ and $\bar{P}_e(\mathcal{B})$, respectively. We design the code $\mathbf{X}(\mathbf{a}, \mathbf{b})$, such that a certain tuple (R_a, D_a, R_b, D_b) of rates and diversities are achievable, where $R_a = R(\mathcal{A}) = \frac{\log(|\mathcal{A}|)}{T}$, $R_b = R(\mathcal{B}) = \frac{\log(|\mathcal{B}|)}{T}$ and analogous to Zheng and Tse, 2003 we define

$$D_a = \lim_{SNR \to \infty} \frac{\log \bar{P}_e(\mathcal{A})}{\log(SNR)}, \; D_b = \lim_{SNR \to \infty} \frac{\log \bar{P}_e(\mathcal{B})}{\log(SNR)}. \tag{1.24}$$

For *fixed rate* codes it has been shown in Diggavi et al., 2004a that to guarantee the diversity orders D_a, D_b we need to design codes such that,

$$\min_{\mathbf{a}_1 \neq \mathbf{a}_2 \in \mathcal{A}} \min_{\mathbf{b}_1, \mathbf{b}_2 \in \mathcal{B}} rank(\mathbf{B}(\mathbf{x}_{\mathbf{a}_1, \mathbf{b}_1}, \mathbf{x}_{\mathbf{a}_2, \mathbf{b}_2})) \geq D_a/M_r \quad (1.25)$$

$$\min_{\mathbf{b}_1 \neq \mathbf{b}_2 \in \mathcal{B}} \min_{\mathbf{a}_1, \mathbf{a}_2 \in \mathcal{A}} rank(\mathbf{B}(\mathbf{x}_{\mathbf{a}_1, \mathbf{b}_1}, \mathbf{x}_{\mathbf{a}_2, \mathbf{b}_2})) \geq D_b/M_r. \quad (1.26)$$

where $\mathbf{B}$ is the codeword difference matrix as defined in (1.7). Basically, this implies that if we transmit a particular message $\mathbf{a} \in \mathcal{A}$, regardless of which message is chosen in message set $\mathcal{B}$, we are ensured a diversity level of D_a for this message set. A similar argument holds for message set $\mathcal{B}$. Using this criterion several diversity-embedded codes have been constructed and will be discussed in the sequel. This design rule is a generalization of the design rule for traditional space-time codes given in Section 3.1.

Linear diversity embedded codes

In Diggavi et al., 2004a, linear constructions of diversity embedded codes were given. These code designs are linear over the complex field in order to be able to decode them efficiently using the *sphere decoder* algorithm Damen et al., 2000 which has an average complexity that is only polynomial (not exponential) in the rate making it an attractive choice for decoding high-rate codes. Another constraint that we impose in our code designs is *not expand the transmitted signal constellation* in contrast with other design based on constellation rotations. For illustration we focus on one code example given in Diggavi et al., 2004a.

Code Example

Let $\mathcal{A}$ come from the message set $\{a(0), a(1)\} \in \mathcal{S}$ and $\mathcal{B}$ come from $\{b(0), b(1), b(2), b(3)\} \in \mathcal{S}$. Hence, $R_a = \frac{1}{2} \log |\mathcal{S}|$, and $R_b = \log |\mathcal{S}|$, leading to a total rate of $R_a + R_b = \frac{3}{2}\log|\mathcal{S}|$.

$$\mathbf{X} = \mathbf{X_a} + \mathbf{X_b} = \begin{bmatrix} a_1 & a_2 & b_3 & b_4 \\ -a_2^* & a_1^* & b_4^* & -b_3^* \\ b_1 & b_2 & a_1^* & -a_2 \\ -b_2^* & b_1^* & a_2^* & a_1 \end{bmatrix} \tag{1.27}$$

where $\mathbf{X_a}$ is a function of variables a_1,a_2 and $\mathbf{X_b}$ is a function of variables b_1,b_2,b_3,b_4. This code is linear over the complex field so that it can be decoded using the sphere decoder Damen et al., 2003 where average complexity is polynomial rather than exponential in the rate. The proof that this code achieves diversity 3 for variables a_1, a_2 and diversity 2 for variables b_1,b_2,b_3,b_4 makes essential use of quaternion arithmetic. The code does not require channel knowledge at the transmitter and it outperforms time-sharing schemes Diggavi et al., 2004a. The performance of this code with perfect and estimated CSI over a quasi-static flat-fading Rayleigh channel is depicted in Figure 1.7.

Non-linear diversity embedded codes

Constructions of a class of non-linear diversity embedded codes was given in Diggavi et al., 2005; Calderbank et al., 2004 and we explain the principles behind these constructions here. The basic idea of this class of non-linear codes is based on using rank properties of binary matrices to construct codes in the complex domain with the desired diversity-embedding property. Given two message sets $\mathcal{A}, \mathcal{B}$, they are mapped to the space-time codeword $\mathbf{X}$ as shown below.

$$\mathcal{A}, \mathcal{B} \xrightarrow{f_1} \mathbf{K} = \begin{bmatrix} K(1,1) & \dots & K(1,T) \\ \vdots & \vdots & \vdots \\ K(M_t,1) & \dots & K(M_t,T) \end{bmatrix} \xrightarrow{f_2} \mathbf{X} = \begin{bmatrix} x(1,1) & \dots & x(1,T) \\ \vdots & \vdots & \vdots \\ x(M_t,1) & \dots & x(M_t,T) \end{bmatrix}$$

where $K(m,n) \in \{0,1\}^{\log(|\mathcal{S}|)}$ *i.e.,* binary string and $x(m,n) \in \mathcal{S}$. This construction is illustrated in Figure 1.9 for a constellation size of L bits.

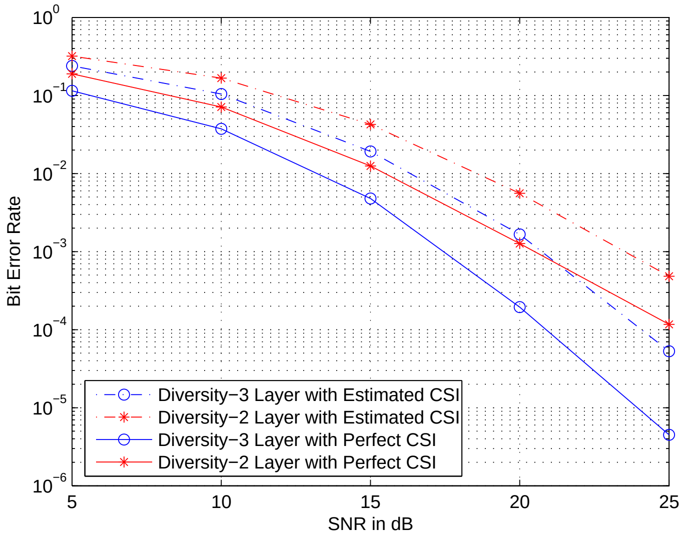


Figure 1.7. Performance of Diversity-Embedding Space-Time Block Code with Perfect and Estimated CSI

The basic idea is that we choose sets $\mathcal{K}_1, \ldots, \mathcal{K}_L$ from which the sequence of binary matrices which encode the constellations are chosen. These sets of binary matrices are chosen so that they have given rank guarantees which reflect the ultimate diversity orders required for each message set. Given the diversity order requirements we can choose these sets appropriately. For example, if we desire a single diversity order (*i.e.*, no diversity embedding) then we can choose all the sets of binary matrices to be identical. At the other extreme all the sets could be different, yielding L different levels of diversity embedding. Given the message set, we first choose the matrices $\mathbf{K}_1, \ldots. \mathbf{K}_L$. The first mapping f_1 is obtained by taking matrices and constructing the matrix $\mathbf{K} \in \mathbb{C}^{M_t \times T}$ each of whose entries is constructed by concatenating the bits from the corresponding entries in the matrices $\mathbf{K}_1, \ldots. \mathbf{K}_L$ into L-length bit-string.

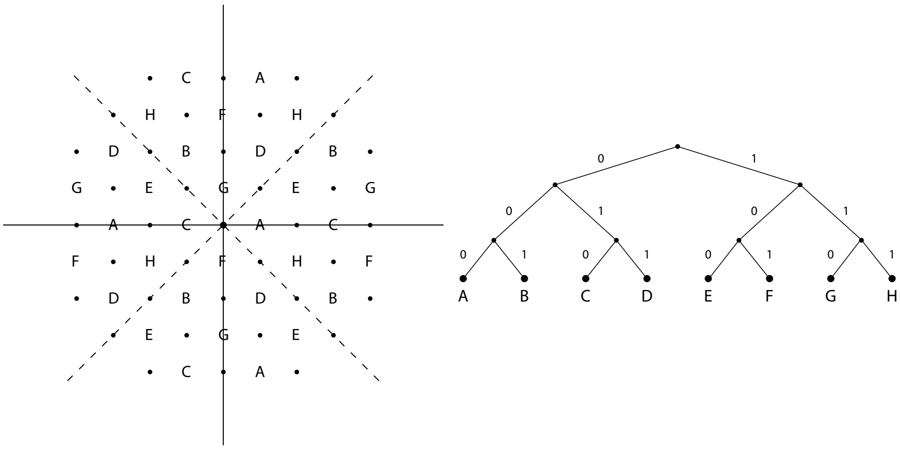


Figure 1.8. A Binary Partition of a 32-Point QAM Constellation

This matrix is then mapped to the space-time codeword through a constellation mapper f_2. This can be done by using a L-level binary partition of a QAM or PSK signal constellation (see Figure 1.8).

As shown above, this structure can be used for one to L levels of diversity order. However, for simplicity, we restrict our attention to two levels of diversity (as shown in Figure 1.6). For concreteness consider a 4-QAM constellation, with 2 levels of diversity order, D_a, D_b. Given that $L = 2$, we then assign layer 1 to diversity order D_a and layer 2 to D_b with $D_a \geq D_b$. We choose sets of binary matrices $\mathcal{K}_1, \mathcal{K}_2$ with rank guarantees $D_a/M_r, D_b/M_r$ respectively. Let the set sizes be $|\mathcal{K}_1| = 2^{TR_a}, |\mathcal{K}_2| = 2^{TR_b}$, yielding the appropriate rates R_a, R_b. Therefore, given a message $m_a \in \mathcal{A}, m_b \in \mathcal{B}$, we choose the matrices $\mathbf{K}_1 \in \mathcal{K}_1, \mathbf{K}_2 \in \mathcal{K}_2$ corresponding respectively to the messages m_a, m_b. Given $\mathbf{K}_1, \mathbf{K}_2$, we can construct the space-time code $\mathbf{X}$ as illustrated in Figure 1.9. If we have constellations of size 2^L with $L > 2$ and we still need 2 levels of diversity, we can assign layers $1, \ldots, L_a$ to choose matrices with the *same* binary set $\mathcal{K}_1$ with rank guarantee D_a/M_r and layers, $L_a + 1, \ldots, L$ to choose matrices from the binary set $\mathcal{K}_2$ with rank guarantee

D_b/M_r. By choosing set cardinalities as $|\mathcal{K}_1| = 2^{T\frac{R_a}{L_a}}, |\mathcal{K}_2| = 2^{T\frac{R_b}{L_b}}$, we get the corresponding rates R_a, R_b for the two diversity orders. Therefore as before given message $m_a \in \mathcal{A}, m_b \in \mathcal{B}$, we choose the sequence of matrices $\mathbf{K}_1, \ldots, \mathbf{K}_{L_a} \in \mathcal{K}_1$ based on m_a and matrices $\mathbf{K}_{L_a+1}, \ldots, \mathbf{K}_L \in \mathcal{K}_2$ based on m_b. Using this sequence of L matrices, we obtain the space-time codeword as seen in Figure 1.9.

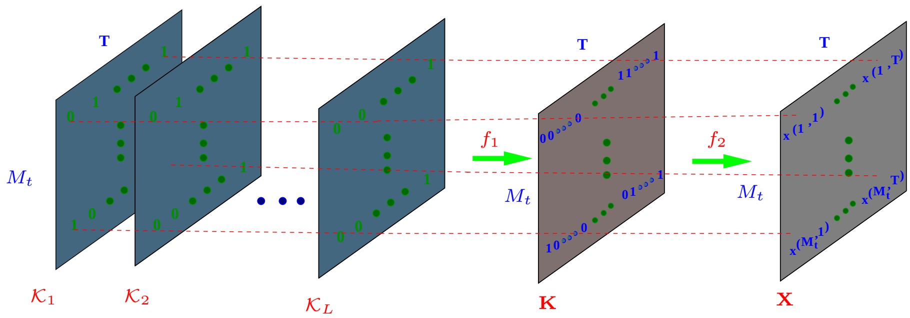


Figure 1.9. Schematic representation of the non-linear code construction.

In all this, the choice of the sets $\mathcal{K}_l, l = 1, \ldots, L$ has been unspecified. However in Lu and Kumar, 2003, sets of $M_t \times T$ binary matrices $\mathcal{P}(M_t, T, r)$ were constructed for $T \geq M_t$ such that the difference of any two matrices in the set had rank $\lfloor M_t - r \rfloor$ over the binary field. They showed that such sets had a cardinality of $|\mathcal{P}(M_t, T, r)| = 2^{T(r+1)}$. Therefore, these matrices yielded a rate of $r + 1$ bits/transmission. In our construction we use these matrices, with $d_l = \lfloor M_t - r_l \rfloor$, with $d_1 \geq d_2 \geq \ldots \geq d_L$. This yields a rate of $R_l = r_l + 1$ in each layer. In Diggavi et al., 2005 it is shown that this construction for QAM constellations achieves the rate tuple $(R_1, M_r d_1, \ldots, R_L, M_r d_L)$, with the overall equivalent single layer code achieving rate-diversity point, $(\sum_l R_l, M_r d_L)$. As described above, we can have the desired number of layers by choosing several identical diversity/rate layers.

The optimal decoding is a maximum-likelihood decoder which jointly decodes the message sets. The performance such a decoder is examined further in Section 4.2 along with applications of diversity embedded codes.

4. Applications

In this section, we show how the STC algebraic structure can be exploited to enhance end-to-end system performance and reduce implementation complexity. This is illustrated through signal processing examples, new STC code constructions, and by examining interactions with the higher networking layers.

4.1 Signal Processing

In this section, we demonstrate how the STC structure can be exploited to reduce the complexity of receiver signal processing algorithms for channel estimation, joint equalization and decoding (both channel-estimate-based and adaptive), and non-coherent detection.

Channel Estimation for Quasi-Static Channels

For quasi-static channels, CSI can be estimated at the receiver using a training sequence embedded in each transmission block. For single-transmit-antenna signalling, the training sequence is only required to have "good" (i.e. impulse-like) auto-correlation properties. However, for the M_t transmit-antenna scenarios, the M_t training sequences should, in addition, have "low" (ideally zero) cross-correlation. In addition, it is desirable (in order to avoid amplifier nonlinear distortion) to use training sequences with constant amplitude. *Perfect Root of Unity Sequences* (PRUS) Chu, 1972 have these ideal correlation and constant-amplitude properties. However, for a given training sequence length, PRUS do not always belong to standard signal constellations such as PSK. Additional challenges in channel estimation for multiple-transmit-antenna systems over the single-transmit-antenna case are the increased number of channel parameters to be estimated and the reduced transmit power (by a factor of M_t) for each transmit antenna.

In Fragouli et al., 2003, it was proposed to encode a single training sequence by a space-time encoder to generate the M_t training sequences[10]. Strictly speaking, this approach is suboptimum since the M_t transmitted training sequences are cross-correlated by the space-time encoder which imposes a constraint on the possible generated training sequences. However, it turns out that, with proper design, the performance loss from optimal PRUS training is negligible Fragouli et al., 2003. Furthermore, this approach reduces the training sequence search space from $(2^b)^{M_t N_t}$ to $(2^b)^{N_t}$ (assuming equal input and output alphabet size 2^b and length-N_t training sequences), making exhaustive searches more practical and thus facilitating the identification of good training sequences from standard signal constellations such as PSK.

The search space can be further reduced by exploiting special characteristics of the particular STC. As an example, consider the 8-state 8-PSK STTC for two transmit and one receive antennas whose equivalent CIR is given by Equation (1.19). For a given transmission block (over which the two channels $h_1(D)$ and $h_2(D)$ are constant), the input sequence determines the equivalent channel. By transmitting only "even" training symbols from the sub-constellation $C_e = \{0, 2, 4, 6\}$, $p_k = +1$ and the equivalent channel is given by $h_e(D) = h_1(D) + Dh_2(D)$. On the other hand, transmitting only "odd" training symbols from the sub-constellation $C_0 = \{1, 3, 5, 7\}$, results in $p_k = -1$ and the equivalent channel $h_o(D) = h_1(D) - Dh_2(D)$. After estimating $h_e(D)$ and $h_o(D)$, we can compute

$$h_1(D) = \frac{h_e(D) + h_o(D)}{2} \quad \text{and} \quad h_2(D) = \frac{h_e(D) - h_o(D)}{2D} . \tag{1.28}$$

[10] We assume, for simplicity, the same space-time encoder for the training and the information symbols. However, they could be different in general.

Consider a training sequence of the form $\mathbf{s} = [\mathbf{s}_e \quad \mathbf{s}_o]$ where $\mathbf{s}_e$ has length $N_t/2$ and takes values in the C_e sub-constellation and $\mathbf{s}_o$ has length $N_t/2$ and takes values in the C_o sub-constellation. Note that if $\mathbf{s}_e$ is a good sequence in terms of MMSE for the estimation of $h_e(D)$, the sequence $\mathbf{s}_o$ created as $\mathbf{s}_o = a\ \mathbf{s}_e$ where $a = exp(\frac{i\pi k}{4})$ and any $k = 1, 3, 5, 7$ achieves the same MMSE for the estimation of $h_o(D)$. Thus, instead of searching over all possible 8^{N_t} sequences $\mathbf{s}$, we can further restrict the search space to the $4^{\frac{N_t}{2}}$ sequences $\mathbf{s}_e$. A reduced-size search can identify sequences $\mathbf{s}_e$ and $\mathbf{s}_o = a\ \mathbf{s}_e$ such that the channel estimation MMSE is achieved. We emphasize that similar reduced-complexity techniques can be developed for other STTCs by deriving their equivalent encoder models (as in Figure 1.3).

In summary, the special STC structure can be utilized to simplify training sequence design for multiple-antenna transmissions without sacrificing performance.

Integration of Equalization and Decoding

Our focus will be on Alamouti-type STBC with two transmit antennas. The treatment can be extended to more than two antennas using orthogonal designs Tarokh et al., 1999a at the expense of some rate loss for complex signal constellations.

The main attractive feature of STBC is the quaternionic structure (see Appendix A for more discussions on quaternions) of the spatio-temporal channel matrix. This allows us to eliminate inter-antenna interference using a low-complexity linear combiner (which is a spatio-temporal matched filter and is also the maximum likelihood detector in this case). Then, joint equalization and decoding for each antenna stream proceeds using any of well-known algorithms for the single-antenna case which can be implemented either in the time or frequency domains. For illustration purposes, we describe next a joint equalization and decoding algorithm for the single-carrier frequency-domain-equalizer (SC

FDE)-STBC. A more detailed discussion and comparison is given in Al-Dhahir, 2002.

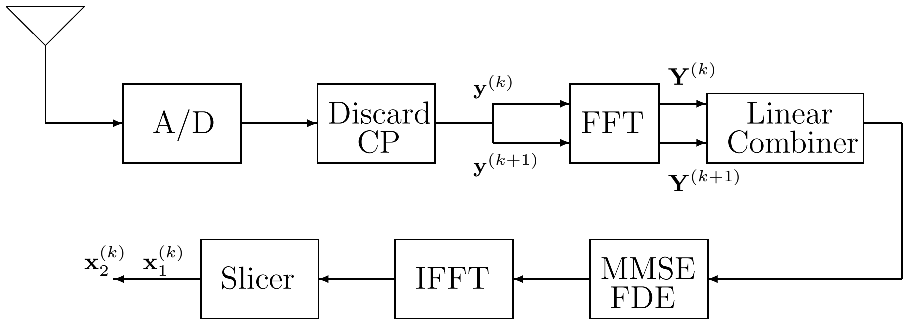


Figure 1.10. FDE-STBC Receiver Block Diagram

The SC FDE-STBC receiver block diagram is given in Figure 1.10. After analog-to-digital (A/D) conversion, the CP part of each received block is discarded. Mathematically, we can express the input-output relationship over the j^{th} received block as follows

$$\mathbf{y}^{(j)} = \mathbf{H}_1^{(j)}\mathbf{x}_1^{(j)} + \mathbf{H}_2^{(j)}\mathbf{x}_2^{(j)} + \mathbf{z}^{(j)} \,, \tag{1.29}$$

where $\mathbf{H}_1^{(j)}$ and $\mathbf{H}_2^{(j)}$ are $N \times N$ circulant matrices whose first columns are equal to $\mathbf{h}_1^{(j)}$ and $\mathbf{h}_2^{(j)}$, respectively, appended by $(N - \nu - 1)$ zeros and $\mathbf{z}^{(j)}$ is the noise vector. Since $\mathbf{H}_1^{(j)}$ and $\mathbf{H}_2^{(j)}$ are circulant matrices, they admit the eigen-decompositions

$$\mathbf{H}_1^{(j)} = \mathbf{Q}^*\mathbf{\Lambda}_1^{(j)}\mathbf{Q} \quad ; \quad \mathbf{H}_2^{(j)} = \mathbf{Q}^*\mathbf{\Lambda}_2^{(j)}\mathbf{Q} \,,$$

where $\mathbf{Q}$ is the orthonormal FFT matrix and $\mathbf{\Lambda}_1^{(j)}$ (resp. $\mathbf{\Lambda}_2^{(j)}$) is a diagonal matrix whose (n,n) entry is equal to the n^{th} FFT coefficient of $\mathbf{h}_1^{(j)}$ (resp. $\mathbf{h}_2^{(j)}$). Therefore, applying the FFT to $\mathbf{y}^{(j)}$, we get (for $j = k, k+1$)

$$\mathbf{Y}^{(j)} = \mathbf{Q}\mathbf{y}^{(j)} = \mathbf{\Lambda}_1^{(j)}\mathbf{X}_1^{(j)} + \mathbf{\Lambda}_2^{(j)}\mathbf{X}_2^{(j)} + \mathbf{Z}^{(j)} \,.$$

The SC FDE-STBC encoding rule is given by Al-Dhahir, 2001

$$\mathbf{X}_1^{(k+1)}(m) = \mathbf{X}_2^{*(k)}(m) \quad \text{and} \quad \mathbf{X}_2^{(k+1)}(m) = -\mathbf{X}_1^{*(k)}(m) \qquad (1.30)$$

for $m = 0, 1, \cdots, N-1$ and $k = 0, 2, 4, \cdots$. The length-N blocks at the FFT output are then processed in pairs resulting in the two blocks (we drop the time index from the channel matrices since they are assumed fixed over the two blocks under consideration)

$$\underbrace{\begin{bmatrix} \mathbf{Y}^{(k)} \\ \mathbf{Y}^{*(k+1)} \end{bmatrix}}_{\mathbf{Y}} = \underbrace{\begin{bmatrix} \mathbf{\Lambda}_1 & \mathbf{\Lambda}_2 \\ -\mathbf{\Lambda}_2^* & \mathbf{\Lambda}_1^* \end{bmatrix}}_{\mathbf{\Lambda}} \underbrace{\begin{bmatrix} \mathbf{X}_1^{(k)} \\ \mathbf{X}_2^{(k)} \end{bmatrix}}_{\mathbf{X}} + \underbrace{\begin{bmatrix} \mathbf{Z}^{(k)} \\ \mathbf{Z}^{*(k+1)} \end{bmatrix}}_{\mathbf{Z}}, \qquad (1.31)$$

where $\mathbf{X}_1^{(k)}$ and $\mathbf{X}_2^{(k)}$ are the FFTs of the information blocks $\mathbf{x}_1^{(k)}$ and $\mathbf{x}_2^{(k)}$, respectively, and $\mathbf{Z}$ is the noise vector. We used the encoding rule in (1.30) to arrive at (1.31). To eliminate *inter-antenna interference*, the linear combiner $\mathbf{\Lambda}^*$ is applied to $\mathbf{Y}$. Due to the quaternionic structure of $\mathbf{\Lambda}$, a second-order diversity gain is achieved. Then, the two decoupled blocks at the output of the linear combiner are equalized separately using the MMSE FDE Sari et al., 1995 which consists of N complex taps per block that mitigate *inter-symbol interference*. Finally, the MMSE-FDE output is transformed back to the time domain using the inverse FFT where decisions are made.

Adaptive Techniques

The coherent receiver techniques described till now require CSI which is estimated and tracked using training sequences/pilot symbols inserted in each block and then used to compute the optimum joint equalizer/ decoder settings. An alternative to this two-step channel-estimate-based approach is *adaptive* space-time equalization/decoding where CSI is not explicitly estimated at the receiver. Adaptive receivers still require training overhead to converge to their optimum settings which, in the presence of channel variations, are adapted using previous decisions to *track* these variations. The celebrated Least Mean Square (LMS) adaptive algorithm Haykin, 1991 is widely used in single-antenna communication systems today due to its low implementation complexity. However, it has been shown to exhibit slow convergence and suffer significant performance degradation (relative to performance achieved with the optimum settings) when applied to broadband MIMO channels due to the large number of parameters that need to be simultaneously adapted and the wide eigenvalue spread problems encountered on those channels. Faster convergence can be achieved by implementing a more sophisticated family of algorithms known as Recursive Least Squares (RLS). However, their high computational complexity compared to LMS and their notorious behavior when implemented in finite precision limit their appeal in practice. It was shown in Younis et al., 2003 that the orthogonal structure of STBC can be exploited to develop fast-converging RLS-type adaptive FDE-STBC at LMS-type complexity. A brief overview is given next.

Our starting point in deriving the adaptive algorithm is the relation

$$\begin{bmatrix} \hat{\mathbf{X}}_1^{(k)} \\ \hat{\mathbf{X}}_2^{(k)} \end{bmatrix} = \begin{bmatrix} \mathbf{A}_1 & \mathbf{A}_2 \\ \mathbf{A}_2^* & -\mathbf{A}_1^* \end{bmatrix} \mathbf{Y} \ , \tag{1.32}$$

where $\mathbf{Y}$ was defined in (1.31) and the diagonal matrices $\mathbf{A}_1$ and $\mathbf{A}_2$ are given by

$$\mathbf{A}_1 = \mathbf{\Lambda}_1^* . diag\{\frac{1}{\tilde{\mathbf{\Lambda}}(i,i) + \frac{1}{SNR}}\}_{i=0}^{N-1} \ ; \ \mathbf{A}_2 = \mathbf{\Lambda}_2^* . diag\{\frac{1}{\tilde{\mathbf{\Lambda}}(i,i) + \frac{1}{SNR}}\}_{i=0}^{N-1} \ , \tag{1.33}$$

with $\tilde{\mathbf{\Lambda}}(i,i) = |\mathbf{\Lambda}_1(i,i)|^2 + |\mathbf{\Lambda}_2(i,i)|^2$. Alternatively, we can write

$$\begin{bmatrix} \hat{\mathbf{X}}_1^{(k)} \\ \hat{\mathbf{X}}_2^{(k)} \end{bmatrix} = \begin{bmatrix} diag(\mathbf{Y}^{(k)}) & -diag(\mathbf{Y}^{*(k+1)}) \\ diag(\mathbf{Y}^{(k+1)}) & diag(\mathbf{Y}^{*(k)}) \end{bmatrix} \begin{bmatrix} \mathbf{W}_1^* \\ \mathbf{W}_2 \end{bmatrix} = \mathbf{U}_k \mathcal{W} \ , \tag{1.34}$$

where $\mathbf{W}_1^*$ and $\mathbf{W}_2$ are vectors containing the diagonal elements of $\mathbf{A}_1^*$ and $\mathbf{A}_2$, respectively, and $\mathcal{W}$ is a $2N \times 1$ vector containing the elements of $\mathbf{W}_1^*$ and $\mathbf{W}_2$. The $2N \times 2N$ quaternionic matrix $\mathbf{U}_k$ contains the received symbols for blocks k and $k+1$. Equation (1.34) can be used to develop a frequency-domain block-adaptive RLS algorithm for $\mathbf{W}$ which, using the special quaternionic structure of the problem, can be simplified to the following LMS-type recursions (see Younis et al., 2003 for details of the derivation)

$$\mathcal{W}_{k+2} = \mathcal{W}_k + \begin{bmatrix} \mathbf{P}_{k+2} & \mathbf{0} \\ \mathbf{0} & \mathbf{P}_{k+2} \end{bmatrix} \mathbf{U}_{k+2}(\mathbf{D}_{k+2} - \mathbf{U}_{k+2}\mathcal{W}_k) \ , \tag{1.35}$$

where $\mathbf{D}_{k+2} = \begin{bmatrix} \mathbf{X}_1^{(k+2)} & \mathbf{X}_2^{*(k+2)} \end{bmatrix}^T$ for the training mode and $\mathbf{D}_{k+2} = \begin{bmatrix} \hat{\mathbf{X}}_1^{(k+2)} & \hat{\mathbf{X}}_2^{*(k+2)} \end{bmatrix}^T$ for the decision-directed mode. The $N \times N$ diagonal matrix $\mathbf{P}_{k+2}$ is computed by the recursion

$$\mathbf{P}_{k+2} = \lambda^{-1}(\mathbf{P}_k - \lambda^{-1}\mathbf{P}_k\mathbf{\Gamma}_{k+2}\mathbf{P}_k) \ , \tag{1.36}$$

where the diagonal matrices $\mathbf{\Gamma}_{k+2}$ and $\mathbf{\Delta}_{k+2}$ are computed from the recursions

$$\begin{aligned} \mathbf{\Gamma}_{k+2} &= diag(\mathbf{Y}^{(k)})\mathbf{\Delta}_{k+2}diag(\mathbf{Y}^{*(k)}) \\ &\quad + diag(\mathbf{Y}^{(k+1)})\mathbf{\Delta}_{k+2}diag(\mathbf{Y}^{*(k+1)}) \\ \mathbf{\Delta}_{k+2} &= (\mathbf{I}_N + \lambda^{-1}(diag(\mathbf{Y}^{(k)})\mathbf{P}_k diag(\mathbf{Y}^{*(k)}) \\ &\quad + diag(\mathbf{Y}^{(k+1)})\mathbf{P}_k diag(\mathbf{Y}^{*(k+1)})))^{-1} \ . \end{aligned}$$

The initial conditions are $\mathcal{W}_0 = \mathbf{0}$, $\mathbf{P}_0 = \delta\mathbf{I}_N$ where δ is a large number, and the forgetting factor λ is chosen close to 1.

The block diagram of the adaptive FDE-STBC is shown in Figure 1.11. Pairs of consecutive received blocks are transformed to the frequency domain using the FFT, then the data matrix in (1.34) is formed. The filter output (the product $\mathbf{U}_k\mathcal{W}_{k-2}$) is transformed back to the time domain using IFFT and passed to a decision device to generate data estimates. The output of the adaptive equalizer is compared to the desired response to generate an error vector which is in turn used to update

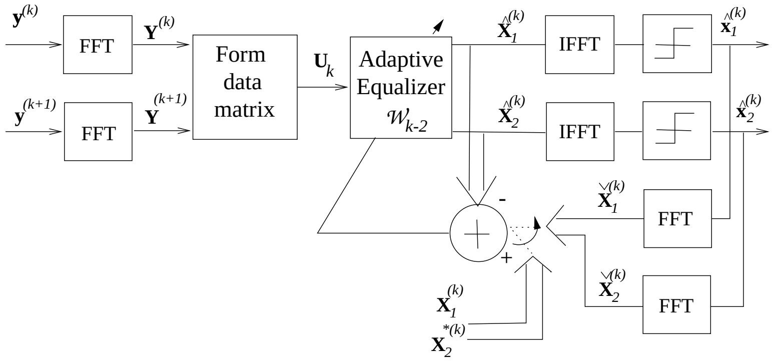


Figure 1.11. Block Diagram of Adaptive FDE-STBC Joint Equalizer/Decoder

the equalizer coefficients according to the RLS recursions. The equalizer operates in a training mode until it converges, then it switches to a decision-directed mode where previous decisions are used for tracking. When operating over fast time-varying channels, retraining blocks can be transmitted periodically to prevent equalizer divergence (see Younis et al., 2003).

Non-Coherent Techniques

Non-coherent transmission schemes do not require channel estimation hence eliminating the need for bandwidth-consuming training sequences and reducing terminal complexity. This becomes more significant for rapidly-fading channels where frequent re-training is needed to track channel variations and for multiple-antenna broadband transmission scenarios where more channel parameters (several coefficients for each transmit-receive antenna pair) need to be estimated. One class of non-coherent techniques are blind identification and detection schemes. Here, the structure of the channel (finite impulse response), the input constellation (finite alphabet) and the output (cyclostationarity) are exploited to eliminate training symbols. Such techniques have a vast literature and we refer the interested reader to a good survey in Tong and

Perreau, 1998. Another class of non-coherent techniques is the generalized ML receiver in Uysal et al., 2001.

Several non-coherent space-time transmission schemes have been proposed for flat-fading channels including differential STBC schemes with two (Tarokh and Jafarkhani, 2000) or more (Jafarkhani and Tarokh, 2001) transmit antennas and group differential STC schemes (see e.g. Hughes, 2000 and references therein). Here, we describe a differential space-time transmission scheme for frequency-selective channels we recently proposed in Diggavi et al., 2002 that achieves full diversity (spatial and multipath) at rate one[11] with two transmit antennas. This scheme is a differential form for the OFDM-STBC structure described in Liu et al., 1999. A time-domain differential space-time scheme with single-carrier transmission is presented in Diggavi et al., 2002.

We consider two symbols $X_1(m)$ and $X_2(m)$ drawn from a PSK constellation which, in a conventional OFDM system, would be transmitted over two consecutive OFDM blocks on the same subcarrier m. Following the Alamouti encoding scheme, the two source symbols are mapped as

$$\mathbf{X}^{(1)}(m) = [X_1(m), X_2(m)]^T, \;\; \mathbf{X}^{(2)}(m) = [-X_2^*(m), X_1^*(m)]^T \;, \quad (1.37)$$

where $\mathbf{X}^{(1)}$ represents the information-bearing vector for the first OFDM block and $\mathbf{X}^{(2)}$ corresponds to the second OFDM block[12]. Let N denote the FFT size, then $\mathbf{X}^{(1)}$ and $\mathbf{X}^{(2)}$ are length-$2N$ vectors holding the symbols to be transmitted by the two transmit antennas. Consequently,

[11]This does not include the rate penalty incurred by concatenating OFDM-STBC with an outer code and interleaving across tones which is common to all OFDM systems (see e.g. Sari et al., 1995 for more discussion.)

[12]Intuitively, each OFDM subcarrier can be thought of as a flat-fading channel and the Alamouti code is applied to each of the OFDM subcarriers. As a result, the Alamouti code yields diversity gains at every subcarrier.

after taking the FFT at the receiver, we have (at subcarrier m)

$$\begin{pmatrix} Y_1(m) & Y_2(m) \\ -Y_2^*(m) & Y_1^*(m) \end{pmatrix} = \begin{pmatrix} H_1(m) & H_2(m) \\ -H_2^*(m) & H_1^*(m) \end{pmatrix} \begin{pmatrix} X_1(m) & -X_2^*(m) \\ X_2(m) & X_1^*(m) \end{pmatrix} + \text{noise}\,, \quad (1.38)$$

where $H_1(m)$ and $H_2(m)$ are the frequency responses of the two channels at subcarrier m.

For block k and subcarrier m, denote the source symbols as $\mathbf{u}_m^{(k)} = \begin{bmatrix} u_{1,m}^{(k)} & u_{2,m}^{(k)} \end{bmatrix}^T$, the transmitted matrix as $\mathbf{X}_m^{(k)}$, and the received matrix as $\mathbf{Y}_m^{(k)}$. Then, in the absence of noise, (1.38) is written as $\mathbf{Y}_m^{(k)} = \mathbf{H}_m \mathbf{X}_m^{(k)}$, where we assume that the channel is fixed over two consecutive blocks. Using the quaternionic structure of $\mathbf{H}_m$, it follows that

$$\mathbf{Y}_m^{*(k-1)}\,\mathbf{Y}_m^{(k)} = (|H_1(m)|^2 + |H_2(m)|^2)\,\mathbf{X}_m^{*(k-1)}\,\mathbf{X}_m^{(k)}\,.$$

Since we would like to estimate the source symbols contained in $\mathbf{U}_m^{(k)} \overset{def}{=} \begin{pmatrix} u_{1,m}^{(k)} & -u_{2,m}^{*(k)} \\ u_{2,m}^{(k)} & u_{1,m}^{*(k)} \end{pmatrix}$, we define the differential transmission rule $\mathbf{X}_m^{(k)} = (\mathbf{X}_m^{*(k-1)})^{-1}\,\mathbf{U}_m^{(k)}$. Note that no inverse computation is needed in computing $(\mathbf{X}_m^{*(k-1)})^{-1}$ due to the quaternionic structure of $\mathbf{X}_m^{*(k-1)}$. Figure 1.12 illustrates the 3 dB SNR loss of differential OFDM-STBC relative to its coherent counterpart (with perfect CSI assumed) for an indoor wireless environment.

4.2 Applications of diversity embedded codes

Given that we can construct diversity embedded codes, we would like to examine how such codes can impact the wireless communication system design. We follow Diggavi et al., 2005 in examing three applications of diversity embedded codes: (i) A natural application would be for applications requiring unequal error protection (UEP). For example,

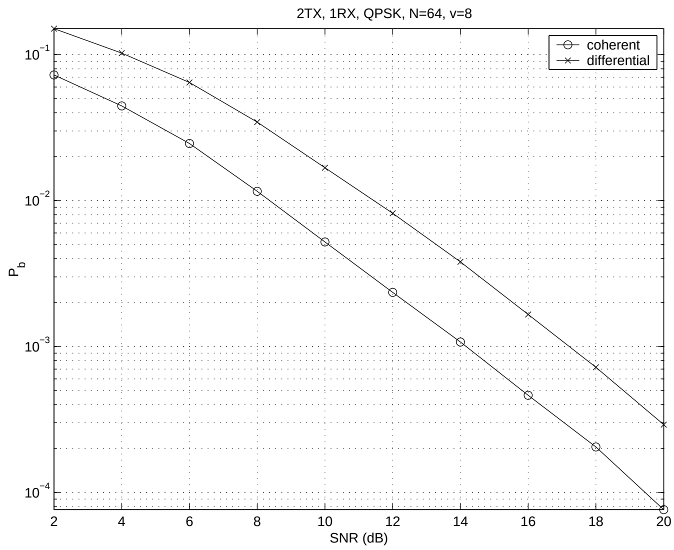
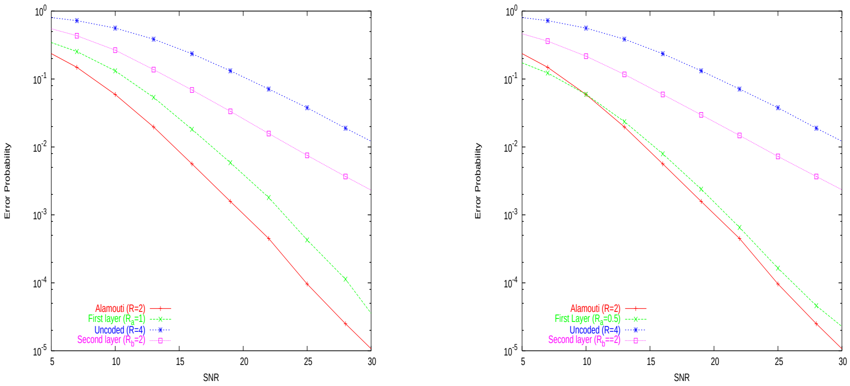


Figure 1.12. Performance Comparison Between Coherent and Differential OFDM-STBC with 2 TX, 1RX, QPSK Modulation, FFT Size of 64, $\nu = 8$

image, audio or video transmission might need multiple levels of error protection for sensitive and less sensitive parts of the message. (ii) A second application could be to improve the overall throughput by opportunistically using the good channel realizations without channel state feedback. (ii) A third application could be in reducing delay in packet transmission using the different diversity orders for prioritized scheduling. In this section, we examine the impact of diversity embedding for each of these applications by comparing it to conventional single layer codes. In all the numerical results we have $M_r = 1$. The numerical results given below are from Diggavi et al., 2005.

In Figure 1.13, the performance of a diversity embedded code which is designed for $M_t = 2$ and 4-QAM signal constellation is given. In Figure 1.13(a), the embedded code has $R_a = 1$ bit/transmission at diversity order $D_a = 2$ and $R_b = 2$ bits/transmission at diversity order $D_b = 1$. We compare this with a "full-rate", maximal diversity order code

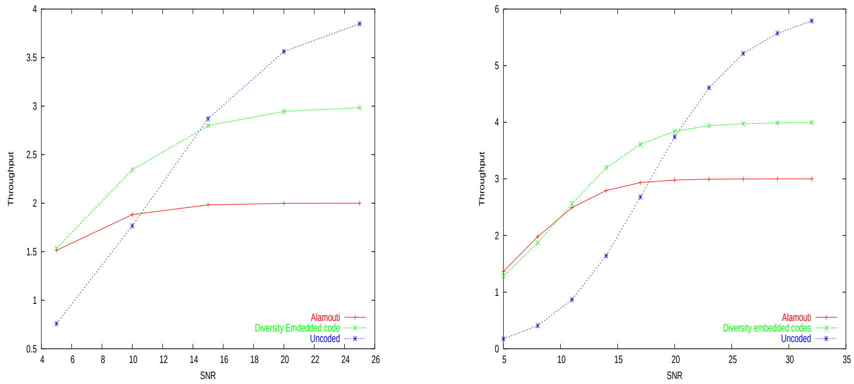


(a) (b)

Figure 1.13. UEP performance of diversity embedded codes.

(Alamouti code with $R = 2$ bits/transmission and diversity order $D = 2$). We also plot the performance of uncoded transmission with rate $R = 4$ bits/transmission and diversity order $D = 1$. Qualitatively, we can see that the embedded code gives two levels of diversity. Note that as expected, we do pay a penalty in rate (or error performance) over a single-layer code designed for the specific diversity order, but the penalty can be made smaller by cutting the rate for one of the diversity layers as demonstrated in Figure 1.13(b).

In Figure 1.14, illustrates the advantage of diversity embedded codes in terms of opportunistically utilizing the channel conditions without feedback. In Figure 1.14(a), we see that for 4-QAM, and $M_t = 2$, the diversity embedded code outperforms the Alamouti single-layer code designed for full diversity. However, at very high SNR, single-layer transmission designed at the lower diversity order outperforms a diversity embedded code since it transmits at a higher rate. This illustrates that for moderate SNR regimes, diversity embedded codes outperform single layer codes in terms of average throughput. Figure 1.14(b) for 8-QAM shows that this regime increases with constellation size.



(a) 4-QAM (b) 8-QAM

Figure 1.14. Throughput comparison of diversity embedded codes with single layer codes.

In the final application given in Diggavi et al., 2005, the delay behavior if we combine a rudimentary ACK/NACK feedback about the transmitted information along with space-time codes, is examined. In the single-layer code the traditional ARQ protocol is used wherein if a packet is in error, it is re-transmitted. In a diversity-embedded code, since different parts of the information get unequal error protection, we can envisage an alternative use of the ARQ. For two diversity levels, we assume that ACK/NACK is received separately for each diversity layer. The mechanism proposed in Diggavi et al., 2005 is illustrated in Figure 1.15. The information is sent along two streams, one on the higher diversity level and the other on the lower diversity level. If the packet on the higher diversity level goes through but the lower diversity level fails, then in the next transmission, the failed packet is sent on the higher diversity level and therefore receives a higher "priority". Therefore the lower priority packet opportunistically rides along with the higher priority packet thereby opportunistically uses the channel to reduce the delay.

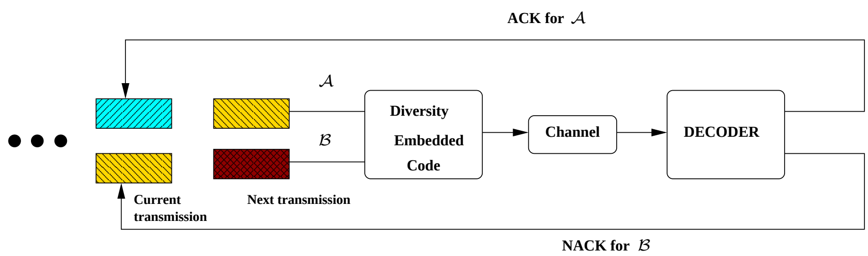


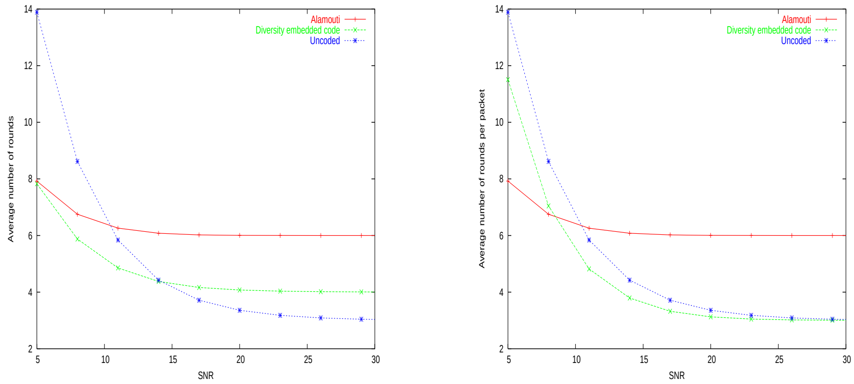
Figure 1.15. ARQ mechanism for diversity embedded codes using prioritized scheduling.

In Figure 1.16, we examine the impact of the ARQ mechanism illustrated in Figure 1.15 with a comparison to single layer schemes. In Figure 1.16(a) we transmit both the single-layer and the diversity embedded code using the same transmit 4-QAM alphabet. We assume that the diversity embedded code gets ACK/NACK feedback on both levels separately. Figure 1.16(b) illustrates the same principle, but with 8-QAM for the diversity embedded code, and 4-QAM for the single layer code. The single-layer scheme at the lower diversity order ($D = 1$) has double the rate of the maximal diversity single-layer code. The comparison is made for the same packet size and therefore it gets individual ACK/NACK for its packets. Figure 1.16 shows that there is an SNR regime where diversity embedded codes give lower average delay than the single layer codes. Qualitatively this is similar to the throughput maximization of Figure 1.14.

4.3 Interactions with network layers

Multiple Access : Interference Cancellation

Spatial diversity implies that different users expect to see different channel conditions. We can double the number of STBC users (and hence network capacity) by adding a second receiver at the base station and employing interference cancellation techniques. We can also deliver higher rates by multiplexing parallel data streams, and in previous work



(a) 4-QAM

(b) 4 and 8-QAM

Figure 1.16. Delay comparison of diversity embedded codes with single layer codes when used with ARQ. In Figure (b) single layer code uses 4-QAM whereas the diversity embedded code uses 8-QAM with rate 2 bits/transmission for each diversity level.

we have described how to use 4 antennas at the base station and 2 antenna at the mobile to deliver twice the standard data rate on a GSM channel.

Our approach is to use algebraic structure to design a single receiver architecture that cancels interference when it is present and delivers increased diversity gain when it is not. We illustrate this for the Alamouti code by showing that a second antenna at the receiver can separate two users, each employing the Alamouti code. Consider vectors $\mathbf{r}_1, \mathbf{r}_2$ where the entries of $\mathbf{r}_i$ are the signals received at antenna i over two consecutive time slots. If $\mathbf{c} = (c_1, c_2)$ and $\mathbf{s} = (s_1, s_2)$ are the codewords transmitted by the first and second users, then

$$\mathbf{r} = \begin{bmatrix} \mathbf{r}_1 \\ \mathbf{r}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{H}_1 & \mathbf{G}_1 \\ \mathbf{H}_2 & \mathbf{G}_2 \end{bmatrix} \begin{bmatrix} \mathbf{c} \\ \mathbf{s} \end{bmatrix} + \begin{bmatrix} \mathbf{w}_1 \\ \mathbf{w}_2 \end{bmatrix}$$

where the vectors $\mathbf{w}_1$ and $\mathbf{w}_2$ are complex Gaussian random variables with zero mean and covariance $N_0\mathbf{I}_2$. The matrices $\mathbf{H}_1$ and $\mathbf{H}_2$ capture the path gains from the first user to the first and second receive antennas. The matrices $\mathbf{G}_1$ and $\mathbf{G}_2$ capture the path gains from the second user to the first and second receive antennas. What is important is that all these matrices share the Alamouti structure. Define

$$\mathbf{D} = \begin{bmatrix} \mathbf{I}_2 & -\mathbf{G}_1\mathbf{G}_2^{-1} \\ -\mathbf{H}_2\mathbf{H}_1^{-1} & \mathbf{I}_2 \end{bmatrix}$$

and observe that

$$\mathbf{Dr} = \begin{bmatrix} \mathbf{H} & \mathbf{0} \\ \mathbf{0} & \mathbf{G} \end{bmatrix} \begin{bmatrix} \mathbf{c} \\ \mathbf{s} \end{bmatrix} + \begin{bmatrix} \tilde{\mathbf{w}}_1 \\ \tilde{\mathbf{w}}_2 \end{bmatrix},$$

where $\mathbf{H} = \mathbf{H}_1 - \mathbf{G}_1\mathbf{G}_2^{-1}\mathbf{H}_2$ and $\mathbf{G} = \mathbf{G}_2 - \mathbf{H}_2\mathbf{H}_1^{-1}\mathbf{G}_1$.

The matrix $\mathbf{D}$ transforms the problem of joint detection of two co-channel users into separate detection of two space-time users. It plays the role of the decorrelating detector in CDMA systems; detection of the codeword $\mathbf{c}$ is through projection onto the orthogonal complement of $[\mathbf{G}_1^T, \mathbf{G}_2^T]$. The algebraic structure of the Alamouti code (closure under addition, multiplication and taking inverses) implies that the matrices $\mathbf{H}$ and $\mathbf{G}$ have the same structure as $\mathbf{H}_1, \mathbf{H}_2, \mathbf{G}_1$, and $\mathbf{G}_2$. Next we show how the algebraic structure of the Alamouti code leads to a single receiver structure that cancels interference when it is present and delivers increased diversity gain when it is not. The covariance matrix $\mathbf{M}$ of the received signal is given by

$$\mathbf{M} = \mathrm{E}[\mathbf{rr}^*] = \underbrace{\begin{bmatrix} \mathbf{H}_1 \\ \mathbf{H}_2 \end{bmatrix} \begin{bmatrix} \mathbf{H}_1^* & \mathbf{H}_2^* \end{bmatrix}}_{\substack{\text{orthogonal proj}^n \\ \text{on } \langle \mathbf{h}_1, \mathbf{h}_2 \rangle}} + \underbrace{\begin{bmatrix} \mathbf{G}_1 \\ \mathbf{G}_2 \end{bmatrix} \begin{bmatrix} \mathbf{G}_1^* & \mathbf{G}_2^* \end{bmatrix}}_{\substack{\text{orthogonal proj}^n \\ \text{on } \langle \mathbf{g}_1, \mathbf{g}_2 \rangle}} + \frac{1}{\mathrm{SNR}}\mathbf{I}_4$$

and if

$$\begin{bmatrix} \mathbf{H}_1 \\ \mathbf{H}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{h}_1 & \mathbf{h}_2 \end{bmatrix} \text{ and } \begin{bmatrix} \mathbf{G}_1 \\ \mathbf{G}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{g}_1 & \mathbf{g}_2 \end{bmatrix}$$

then it can be shown that if $i \neq j$ then for all integers k, we have $\mathbf{h}_i\mathbf{M}^k\mathbf{h}_j^* = \mathbf{g}_i\mathbf{M}^k\mathbf{g}_j^* = 0$ (see Section 4 of Calderbank and Naguib, 2001).

The MMSE receiver looks for a linear combination $\alpha^*\mathbf{r}$ of received signals that is close to some linear combination $\beta_1 c_1 + \beta_2 c_2$ of the codeword $\mathbf{c}$. The solution turns out to be

$$\alpha_1 = (\mathbf{M} - \mathbf{h}_2\mathbf{h}_2^*)^{-1}\mathbf{h}_1; \qquad \beta_1 = 1, \qquad \beta_2 = \frac{\mathbf{h}_2^*\mathbf{M}^{-1}\mathbf{h}_1}{1 - \mathbf{h}_2^*\mathbf{M}^{-1}\mathbf{h}_1}$$

$$\alpha_2 = (\mathbf{M} - \mathbf{h}_1\mathbf{h}_1^*)^{-1}\mathbf{h}_2; \qquad \beta_2 = 1, \qquad \beta_1 = \frac{\mathbf{h}_1^*\mathbf{M}^{-1}\mathbf{h}_2}{1 - \mathbf{h}_1^*\mathbf{M}^{-1}\mathbf{h}_2}$$

Either $\beta_1 = 0$ and $\beta_2 = 1$ or $\beta_2 = 0$ and $\beta_1 = 1$! The MMSE interference canceller maintains the separate detection feature of space-time block codes; errors in decoding $\mathbf{c}_1$ do not influence the decoding of $\mathbf{c}_2$ and vice versa. Generalizations to the case of frequency-selected channels are described in Diggavi et al., 2003b.

Integration of Physical, Link, and Transport Layers

It is well-known that errors at the wireless physical layer reverberate across layers and have a negative impact on TCP performance (see Balakrishnan et al., 1997 and references therein).

Roughly speaking, TCP interprets frame/packet losses as signs of network congestion, and cuts the transmission rate by half, whenever these error events occur. When the link layer does not hide frame errors, TCP times out and does not transmit anything for a significant amount of time. Figure 1.17 (see the discussion in Stamoulis and Al-Dhahir, 2003) shows the ability of space-time block codes (in this case the Alamouti code) to shift the SNR point at which TCP breaks down resulting in significant improvements in throughput. In this example, there are no frame retransmissions at the link-layer, and the operating point is in an SNR region where the BER performance with only one transmit antenna is below the TCP breaking-point threshold, and the BER with space-time codes is above this particular threshold.

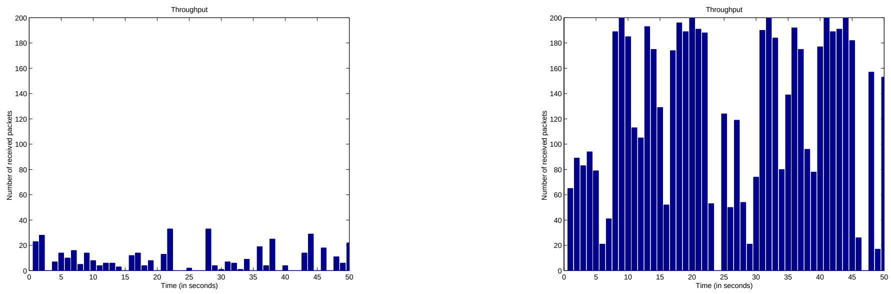


Figure 1.17. TCP Throughput with One (Left Figure) and Two (Right Figure) Transmit Antennas

Network Utility Maximization (NUM)

This brief section provdes a glimpse into an emerging foundation for networking that is both mathematically rich and pratically relevant, with a promising track record of impact on commercial systems (for a survey see Chiang et al., 2006).

Layered architecture is one of the most fundamental and influential structure of network design. Each layer in the protocol stack hides the complexity of the layer below and provides a service to the layer above. While the general principle of layering is widely recognized as one fo the key reasons for the enormous success of the Internet, there is little quantitative understanding on a systematic, rather than an ad hoc, process of designing layered protocol stack for wired and wireless networks. One possible perspective to rigorously and holistically understand layering is to integrate the various protocol layers into a single coherent theory, by regarding them as carrying out an asynchronous distributed computation over the network to implicitly solve a global objective. Such a theory will expose the interconnection between protocol layers and can be used to study rigorously the performance tradeoff in protocol layering, as different ways to distribute a centralized computation. Even though the design of a complex system will always be broken down into simpler modules, this theory will allow us to systematically carry out this layering process and explicitly trade off design objectives.

The approach of 'protocol as a distributed solution' to some global optimization problem in the form of NUM has been succesfully tested in trials for TCP. The key innovation from this line of work is to view network as an optimization solver and congestion control protocol as distributed algorithms solving a specified NUM. The framework of NUM has recently been substantially extended from an analytic tool of reverse-engineering TCP congestion control to a general approach of understanding interactions across layers. Application needs form the objective function, i.e., network utility to be maximized, and the restrictions in the communication infrastructure are translated into many constraints of a generalized NUM problem. Such problems may be very difficult nonlinear, nonconvex optimization with integer constraints. There are many different ways to decompose a given problem, each of which corresponds to a different layering scheme. These decomposition (i.e., layering) schemes have different trade-offs in efficiency, robustness, and asymmetry of information and control, thus some are 'better' than others depending on the criteria set by the network users and managers.

The key idea in 'layering as optimization decomposition' is as follows. Different *decompositions* of an optimization problem, in the form of a generalized NUM are mapped to different *layering* schemes in a communication network, and from functions of primal or Lagrange dual *variables* coordinating the subproblems to the *interfaces* among the layers. Since different decompositions correspond to different layer architectures, we can also tackle the question 'how to and how not to layer' by investigating the pros and cons of decomposition techniques. Furthermore, by comparing the objective function values under various forms of optimal decompositions and suboptimal decompositions, we can seek 'separation theorems' among layers: conditions under which strict layering incurs no loss of optimality. Robustness of these separation theorems can be further characterized by sensitivity analysis in optimization theory: how much will the differences in the objective value (between different layer schemes) fluctuate as constant parameters in utility maximization are perturbed.

5. Discussion and Future Challenges

A discussion about the issues and tradeoffs involved in MIMO system design is now in order. These issues include choice of key system parameters including block length, carrier frequency, and number of transmit/receive antennas in addition to the operating environment conditions such as high vs. low SNR, high vs. low mobility, and strict vs. relaxed delay constraints.

The length of the transmission block N (relative to the symbol period and the channel memory ν) is an important design parameter. Shorter blocks experience less channel time variation (which reduces the need for channel tracking within the block), incur smaller delay, and have smaller receiver complexity (typically, block-by-block signal processing algorithm complexity grows in a quadratic or cubic manner with block size). On the other hand, smaller blocks could incur a significant throughput penalty due to overhead (needed for various functions including guard sequence, synchronization, training, etc.).

Concerning the carrier frequency f_c, the current trend is towards higher f_c where more RF bandwidth is available, antenna size is smaller (at the same radiation efficiency), and antenna spacing requirements (to ensure independent fading) are less stringent due to decreased wavelenghth. On the other hand, the main challenges in migrating towards higher f_c are the higher costs of manufacturing reliable RF components, the increased propagation loss, and the increased sensitivity to Doppler effects.

When selecting the number of transmit/receive antennas, several practical considerations must be taken into account, as described next. Under strict delay constraints, achieving high diversity gains (i.e. high reliability) becomes critical in order to minimize the need for retransmissions. Since transmit/receive diversity gains experience diminishing returns as their numbers increase, complexity considerations dictate the use of small antenna arrays (typically no more than 4 antennas at each end). Current technology limitations favor using more antennas at the base station than at the user terminal.

For delay-tolerant applications (such as data file transfers), achieving high throughput takes precedence over achieving high diversity and larger antenna arrays (of course still limited by cost and space constraints) can be used to achieve high spatial rate multiplexing gains. Likewise, high-mobility channel conditions substantially impact the choice of system parameters such as the use of shorter blocks, lower carrier frequencies, and non-coherent or adaptive receiver techniques.

STTC Tarokh et al., 1998a use multiple transmit antennas to achieve diversity and coding gains. The first gain manifests itself as an increase in the slope of the BER vs. SNR curve (on a log-log scale) at high SNR, while the latter gain manifests itself as a horizontal shift in that same curve. At low SNR, it becomes more important to maximize the coding gain while at high SNR diversity gains dominate performance. For SNR ranges typically encountered on broadband wireless terrestrial links, it might be wise to sacrifice some diversity gain in exchange for more coding gain. For example, using only two transmit and one receive antennas for a channel with delay spread as high as 16 taps, the maximum (spatial and multipath) diversity gain possible is $16 \times 2 \times 1 = 32$! For typical SNR levels in the 10-25 dB range, it suffices to design STC's that achieve a much smaller diversity level (e.g. up to 8) to limit the receiver complexity and to use the extra degrees of freedom in code design to achieve a higher coding gain. STC have also been shown to result in significant improvements in the networking throughput Stamoulis and Al-Dhahir, 2003.

Wireless networks present an opportunity to re-examine functional abstractions of traditional network layer protocols. Cross-layer interactions in wireless networks can optimize throughput by making additional performance information visible between layers in the IP protocol stack. Spatial diversity is critical in improving data rates and reliability of individual links and leads to innovations in scheduling that optimize global throughput. Space-time codes designed for small numbers of transmit and receive antennas have been shown to significantly improve link capacity, and also system capacity through resource allocation. This coding technology can be integrated with sophisticated signal processing to provide a complete receiver that has computational complexity essentially implementable on current chip technology. This bounding of signal processing complexity is important given the energy constraints at the mobile terminal.

Many challenges still exist at the physical layer on the road to achieving high rate and reliability wireless transmission. We conclude this chapter by enumerating some of these challenges

- **Signal Processing :** While effective and practical joint equalization and decoding schemes that exploit the multipath diversity available in frequency-selective channels have been developed, the full exploitation of time diversity in fast time-varying channels remains elusive. The main challenge here is the development of practical adaptive algorithms that can track the rapid variations of the large number of taps in MIMO channels and/or equalizers. While some encouraging steps have been made in this direction Komninakis et al., 2002; Younis et al., 2003, the allowable Doppler rates (which depend on the mobile speed and carrier frequency) for high performance are still quite limited.

  Another signal processing challenge is the design of MIMO training sequences that are resilient to practical impairments such as receiver synchronization errors (for example, residual frequency offsets in OFDM). It is also of practical importance to construct training sequences with low peak to average power ratio to extend battery life by improving efficiency of the transmit power amplifier. Some of our recent work in this area is described in Minn and Al-Dhahir, 2005b; Minn and Al-Dhahir, 2005a.

- **Code Design :** One challenging code design problem that has attracted significant interest recently is the design of practical space-time codes that achieve the optimal rate-diversity tradeoff Zheng and Tse, 2002a and have a practical decoding complexity. As mentioned in Section 3.3, some progress has been made in Yao and Wornell, 2003; Elia et al., ; Tavildar and Viswanath, 2006. Another challenging problem is the design of non-coherent encoding/decoding schemes for the family of diversity-embedding codes.

- **Networking :** The interference cancellation techniques described in Section 4.3 can be extended in several directions. Given the importance of mobile ad hoc networks and their lack of fixed infrastructure and centralized control, it would be interesting to drop the assumption of time-synchronous users and explore the asynchronous case. Furthermore, given the commercial interest in wireless systems with four transmit antennas, it is important to explore interference cancellation based on the octonion space-time block code, and on the nonlinear quaternionic code described in Section 3.4.

  Another challenging problem is the investigation of cross-layer interactions between embedded-diversity coding and link-layer Automatic Repeat Request (ARQ) protocols (which come in several hybrid and selective forms). In particular, the reliability that is lost when spatial diversity is traded for rate can be recovered by the time diversity gained through ARQ retransmission. Conversely, when rate is traded for spatial diversity, it would be interesting to quantify the value of reduced latency in terms of throughput, delay, and power consumption.

6. Bibliography

The past decade has witnessed significant progress in the understanding and design of space-time codes. The information-theoretic underpinnings of space-time codes was given in Telatar, 1999, Foschini, 1996, who established that multiple antennas can make wireless communication a high data-rate pipe. Though the rudiments of transmit spatial diversity were proposed in Uddenfeldt and Raith, 1992; Wittneben, 1993, the basis of modern space-time coding was given in Tarokh et al., 1998a and Alamouti, 1998a. Since then, space-time codes were extended through linear (Tarokh et al., 1999a, Hassibi and Hochwald, 2002, El-Gamal and Damen, 2003 and references therein) and non-linear designs (Hammons and El-Gamal, 2000, Lu and Kumar, 2005a and references therein). In another line of work, non-coherent space-time codes, their design, analysis and information-theoretic properties have been studied in Hochwald and Marzetta, 1999; Zheng and Tse, 2002b; Hughes, 2000; Hochwald and Sweldens, 2000 and references therein.

The diversity-multiplexing trade-off was first established in Tarokh et al., 1998a in the context of *fixed* transmit alphabet size. The information-theoretic question was posed and answered in Zheng and Tse, 2003. Since then, there has been a significant effort in designing codes achieving the diversity-multiplexing trade-off (see Yao and Wornell, 2003; Elia et al., ; Tavildar and Viswanath, 2006 and references therein). The idea of diversity embedded space-time codes was first proposed in Diggavi et al., 2004a where design criteria and some constructions were given. Diversity embedding from an information-theoretic viewpoint was examined in Diggavi and Tse, 2005.

There has been extensive work in the area of signal processing techniques for space-time codes (see for example the papers in the special issue Al-Dhahir et al., 2002b and references therein). A more extensive survey of developments in diversity commmunications can also be found in Diggavi et al., 2004b. Several recent textbooks Tse and Viswanath, 2005; Goldsmith, 2005 are excellent introductions to modern wireless communications.

Appendix A
Algebraic Structure : Quadratic Forms

The simplest form of transmit diversity is the delay diversity scheme proposed by Wittneben Wittneben, 1993 for two transmit antennas, where a signal is transmitted from the second antenna, then delayed one time slot and transmitted from the first antenna. Orthogonal designs Tarokh et al., 1999a are a class of space-time block codes that achieve maximal diversity with decoding complexity that is linear in the size of the constellation. The most famous example was discovered by Alamouti Alamouti, 1998b, and is described by a 2×2 matrix where the columns represent different time slots, the rows represent different antennas, and the entries are the symbols to be transmitted. The encoding rule is

$$\begin{bmatrix} c_1 & c_2 \end{bmatrix} \rightarrow \begin{bmatrix} c_1 & c_2 \\ -c_2^* & c_1^* \end{bmatrix}$$

Assuming a quasi-static flat-fading channel, the signals r_1, r_2 received over two consecutive time slots are given by

$$\begin{bmatrix} r_1 \\ -r_2^* \end{bmatrix} = \begin{bmatrix} h_1 & h_2 \\ -h_2^* & h_1^* \end{bmatrix} \begin{bmatrix} c_1 \\ -c_2^* \end{bmatrix} + \begin{bmatrix} w_1 \\ -w_2^* \end{bmatrix}$$

where h_1, h_2 are the path gains from the two transmit antennas to the mobile, and the noise samples w_1, w_2 are independent samples of a zero-mean complex Gaussian random variable with noise energy N_0 per complex dimension. Thus $\mathbf{r} = \mathbf{H}\mathbf{c} + \mathbf{w}$ where the matrix $\mathbf{H}$ is orthogonal. The reason for broad commercial interest in the Alamouti code is that both coherent and non coherent detection are remarkably simple. If the path gains are known at the mobile (typically this is accomplished at some sacrifice in rate by inserting pilot tones into the data frame for channel estimation) then the receiver is able to form

$$\mathbf{H}^*\mathbf{r} = \|\mathbf{h}\|^2\mathbf{c} + \mathbf{w}'$$

The new noise term $\mathbf{w}'$ is still white, so that c_1, c_2 can be decoded separately rather than jointly, which is far more complex.

Let $u_0, u_1, \ldots, u_{s-1}$ be positive integers, and let $x_0, x_1, \ldots, x_{s-1}$ be commuting indeterminates. A *real orthogonal design* of type $(u_0, u_1, \ldots, u_{s-1})$ and size N is an $N \times N$ matrix $\mathbf{X}$ with entries $0, \pm x_0, \pm x_1, \ldots, \pm x_{s-1}$ satisfying

$$\mathbf{X}\mathbf{X}^T = \left(\sum_{j=0}^{s-1} u_j x_j^2\right) \mathbf{I}_N$$

There are s indeterminates and N time slots, so the rate of the orthogonal design is s/N.

N=2: A real orthogonal design of type (1,1) and size $N = 2$ corresponds to the representation of the complex numbers $\mathbf{C}$ as a 2×2 matrix algebra over the real numbers $\mathbf{R}$. The complex number $x_0 + \mathbf{i}x_1$ corresponds to the matrix $\begin{bmatrix} x_0 & x_1 \\ -x_1 & x_0 \end{bmatrix}$.

N=4: A real orthogonal design of type (1,1,1,1) and size $N = 4$ corresponds to the representation of the quaternions $\mathbf{Q}$ as a 4×4 matrix algebra over the real numbers $\mathbf{R}$. The quaternion $x_0 + \mathbf{i}x_1 + \mathbf{j}x_2 + \mathbf{k}x_3$ corresponds to the matrix

$$\left[\begin{array}{cc|cc} x_0 & x_1 & x_2 & x_3 \\ -x_1 & x_0 & -x_3 & x_2 \\ \hline -x_2 & x_3 & x_0 & -x_1 \\ -x_3 & -x_2 & x_1 & x_0 \end{array}\right] = x_0\mathbf{I}_4 + x_1 \left[\begin{array}{cc|cc} & 1 & & \\ -1 & & & \\ \hline & & & -1 \\ & & 1 & \end{array}\right]$$

$$+\, x_2 \left[\begin{array}{cc|cc} & & 1 & \\ & & & 1 \\ \hline -1 & & & \\ & -1 & & \end{array}\right] + x_3 \left[\begin{array}{cc|cc} & & & 1 \\ & & -1 & \\ \hline & 1 & & \\ -1 & & & \end{array}\right]$$

N=8: A real orthogonal design of type (1,1,. . . ,1) and size $N = 8$ corresponds to the representation of the octonions or Cayley numbers as an 8-dimensional algebra over the real numbers $\mathbf{R}$. This algebra is non-associative as well as non-commutative.

A *complex orthogonal design* of size N and type $(u_0, u_1, \ldots, u_{s-1}; v_1, v_2, \ldots, v_t)$ is a matrix $\mathbf{Z} = \mathbf{X} + \mathbf{i}\mathbf{Y}$, where $\mathbf{X}$ and $\mathbf{Y}$ are real orthogonal designs of type $(u_0, u_1, \ldots, u_{s-1})$ and $(v_1, v_2, \ldots, v_t)$ respectively, and where

$$\mathbf{Z}\mathbf{Z}^* = \left(\left(\sum_{j=0}^{s-1} u_j x_j^2\right) + \left(\sum_{j=1}^{t} v_j y_j^2\right)\right) \mathbf{I}_N$$

Since $\mathbf{Z}\mathbf{Z}^* = (\mathbf{X} + \mathbf{i}\mathbf{Y})(\mathbf{X}^T - \mathbf{i}\mathbf{Y}^T) = (\mathbf{X}\mathbf{X}^T + \mathbf{Y}\mathbf{Y}^T) + \mathbf{i}(\mathbf{Y}\mathbf{X}^T - \mathbf{X}\mathbf{Y}^T)$

it follows that $\mathbf{Y}\mathbf{X}^T = \mathbf{X}\mathbf{Y}^T$. A pair of real orthogonal designs that is connected in this way is called an *amicable pair* (see Geramita and Seberry Geramita and Seberry, 1979 for more information). Note that if $t = s$, then the entries of $\mathbf{X} + \mathbf{i}\mathbf{Y}$ are linear combinations of the complex indeterminates $z_k = x_k + \mathbf{i}y_k$ and their complex conjugates $z_k^* = x_k - \mathbf{i}y_k$. In fact, the definition of a complex orthogonal design found in Tarokh et al. Tarokh et al., 1999a is given in terms of these indeterminates. The rate of a complex orthogonal design is $(s+t)/2N$.

A complex design of size N with $t = s+1$ determines a real orthogonal design of size $2N$ through the substitution

$$x_0 + \mathbf{i}x_1 \rightarrow \begin{bmatrix} x_0 & x_1 \\ -x_1 & x_0 \end{bmatrix}.$$

N=2: This is the Alamouti space-time block code. We may view quaternions as pairs of complex numbers, where the product of quaternions (a, b) and (c, d) is given by $(ac - bd^*, ad + bc^*)$. These are Hamilton's Biquaternions, and if we associate the pair (a, b) with the 2×2 complex matrix $\begin{bmatrix} a & b \\ -b^* & a^* \end{bmatrix}$ then we see that the rule for multiplying biquaternions coincides with the rule for matrix multiplication.

N=4: The Alamouti space-time block code determines the full-rate 4×4 real orthogonal design via the above substitutions. However the full rate 8×8 real orthogonal design cannot be obtained from a 4×4 complex design.

The representation of the octonions as 4-tuples of complex numbers provides an example of an extremal complex design. The product $\mathbf{c} = \mathbf{ab}$ of octonions $\mathbf{a} = (a_0, a_1, a_2, a_3)$ and $\mathbf{b} = (b_0, b_1, b_2, 0)$ is given by

$$\begin{aligned} c_0 &= a_0 b_0 - b_1^* a_1 - b_2^* a_2 - a_3^* b_3 \\ c_1 &= b_1 a_0 + a_1 b_0^* - a_3 b_2^* + b_3 a_2^* \\ c_2 &= b_2 a_0 - a_1^* b_3 + a_2 b_0^* + b_1^* a_3 \\ c_3 &= b_3 a_0^* + a_1 b_2 - b_1 a_2 + a_3 b_0 \end{aligned}$$

It follows that right multiplication of an octonion $\mathbf{a}$ by octonions of the form $\mathbf{b} = (b_0, b_1, b_2, 0)$ can be represented as $\mathbf{ab} = \mathbf{a}R(b_0, b_1, b_2, 0)$, where

$$R(b_0, b_1, b_2, 0) = \begin{bmatrix} b_0 & b_1 & b_2 & 0 \\ -b_1^* & b_0^* & 0 & b_2 \\ -b_2^* & 0 & b_0^* & -b_1 \\ 0 & -b_2^* & b_1^* & b_0 \end{bmatrix} \tag{A.1}$$

The columns of this matrix are orthogonal; hence $R(b_0, b_1, b_2, 0)$ is a rate $\frac{3}{4}$ complex orthogonal design.

Given t symmetric, anti-commuting orthogonal matrices of size N, let $\rho_t(N)-1$ be the number of skew-summetric, anti-commuting orthogonal matrices of size N that anti-commute with the initial set of t matrices. The next two theorems are proved using Clifford algebras Clifford, 1878 and are due to Wolfe Wolfe, 1976.

THEOREM A.1 *There exists an amicable pair* $\mathbf{X}, \mathbf{Y}$ *of real orthogonal designs of size* N*, where* $\mathbf{X}$ *has type* $(1, \ldots, 1)$ *on variables* $x_0, x_1, \ldots, x_{s-1}$ *and* $\mathbf{Y}$ *has type* $(1, \ldots, 1)$ *on variables* $y_1, y_2, \ldots, y_t$*, if and only if* $s \leq \rho_t(N) - 1$.

THEOREM A.2 *Let* $\mathbf{X}, \mathbf{Y}$ *be an amicable pair of real orthogonal designs of size* $N = 2^h N_0$ *where* N_0 *is odd. Then the total number of real variables in* $\mathbf{X}$ *and* $\mathbf{Y}$ *is at most* $2h+2$*, and this bound is achieved by designs* $\mathbf{X}, \mathbf{Y}$ *that each involve* $h+1$ *variables.*

In fact, a group of Pauli matrices that appears in the construction of quantum error correcting codes can be used to construct pairs $\mathbf{X}, \mathbf{Y}$ where the entries of $\mathbf{X}$ are $0, \pm x_0, \ldots, \pm x_s$ and the entries of $\mathbf{Y}$ are $0, \pm y_1, \ldots, \pm y_t$ Calderbank and Naguib, 2001.

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