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An Achievable Rate Region for Gaussian Interference Channel with Intermittent Feedback

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Abstract— **We consider the two-user Gaussian interference channel with intermittent channel output feedback. We derive an achievable rate region that corresponds to the capacity region of the linear deterministic version of the problem. The result shows that passive and unreliable feedback can be harnessed to provide multiplicative capacity gain in Gaussian interference channels. In contrast to other schemes developed for interference channel with feedback, our achievable scheme makes use of *quantize-map-and-forward* to relay the information obtained through feedback, performs forward decoding, and does not use structured codes. We find that when the feedback links are active with sufficiently large probabilities, the perfect feedback sum-capacity is achieved to within a constant gap.**

I. Introduction

Feedback has been shown to be a promising strategy for interference management [1]. In contrast to point-to-point memoryless channels, where feedback gives no capacity gain [2], and multiple-access channels, where feedback can at most provide *power* gain [3], Suh and Tse [1] showed that channel output feedback can result in multiplicative gain in Gaussian interference channel (IC) capacity. They considered a model where each transmitter-receiver pair is equipped with a perfect out-of-band feedback link, which provides the transmitter with a noiseless observation of the last channel output of its corresponding receiver. Given the optimistic result obtained under this setting, a natural question arises: Can feedback be leveraged for interference management under more realistic models?

There have been several pieces of work so far, attempting to answer this question. Vahid *et al.* [4] considered a rate-limited feedback model, where the feedback links are modeled as fixed-capacity deterministic bit pipes. They developed a scheme based on decode-and-forward at transmitters and lattice coding to extract the helping information in the feedback links, and showed that it achieves the sum-capacity to within a constant gap. The work in [5] studied a deterministic model motivated by passive feedback over AWGN channels, and [6], [7] studied the two-way interference channel, where the feedback is provided through a backward interference channel that occupies the same resource as the forward channel. [5], [6] and [7] only dealt with the linear deterministic model [8] of the Gaussian IC.

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In this paper, we consider the passive intermittent feedback model for the Gaussian interference channel. The passive intermittent feedback model was introduced in [9] and investigated on the linear deterministic interference channel. In this model, feedback is available to the transmitters through erasure channels controlled by Bernoulli processes $\{S_1[t], S_2[t]\}$. The particular realization of the pair $\{S_1[t], S_2[t]\}$ is available to the users causally. Although the joint distribution $p(S_1[t], S_2[t])$ can be time-variant in general, we focus on an i.i.d. model for simplicity. In our earlier work [9], we characterized the capacity region of the linear deterministic channel under this model; in this work, we extend our result to the Gaussian channel.

The intermittent feedback model can be relevant in several scenarios. If the resource used for feedback is not dedicated, uncoordinated interference or collisions over the feedback channel may cause this resource to be available intermittently at the transmitter. In some other scenarios, packet drops or control mechanisms in higher layers may cause intermittent feedback, if a side-channel such as WiFi is used as a feedback resource.

In addition to intermittence, the other important feature of our feedback model is the *passiveness* of feedback: The receivers simply feedback their channel outputs back to the transmitters without any processing. In other words, each transmitter receives from feedback an observation of the channel output of its own receiver through an erasure channel, with unit delay. We focus on the passive feedback model as the intermittence of feedback is motivated by the availability of feedback resources (either through use of best-effort WiFi for feedback or through feedback resource scheduling). Therefore, it might be that the time-variant statistics of the intermittent feedback are not *a priori* available at the receiver, precluding active coding. Moreover, the availability of the feedback resource may not be known ahead of transmission, therefore motivating the assumption of causal state information at the transmitter. If the receiver has *a priori* information about the feedback channel statistics, it can perform active coding, in which case, the intermittent feedback model reduces to the rate-limited model of [4].

Our achievable scheme has three main differences from the previous schemes developed in [1], [4] and [5]. First, we use *quantize-map-and-forward* [8] at the transmitters to send the information obtained through feedback, as opposed to (partial or complete) decode-and-forward, which has been used in [1], [4], [5]. Second, at the receivers, we perform forward decoding of blocks instead of backward decoding, which results in a better delay performance. Third, we do not use structured codes, *i.e.*, we only perform random coding.

Our result shows that feedback can be harnessed to provide multiplicative gain in Gaussian interference channel capacity even when the feedback is unreliable and intermittent. The derived achievable rate region agrees with the capacity region of the linear deterministic channel. A consequence of this result is that when the feedback links are active with large enough probabilities, the sum-capacity of the perfect feedback channel can be achieved to within a constant gap. This is a direct extension of a similar observation for the linear deterministic case [9]. We also extend our result to parallel vector channels, which can be used as a model for OFDM and packet drops over a best-effort channel.

II. Model

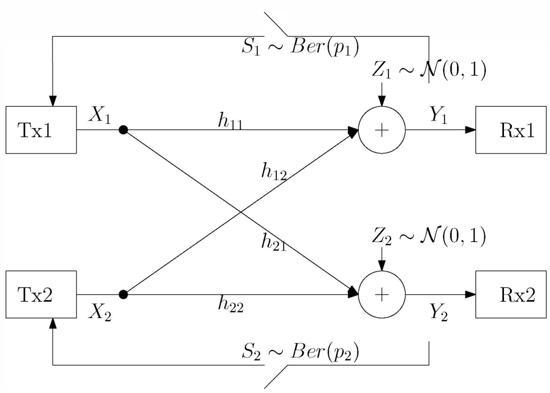

Fig. 1. Two-user Gaussian interference channel with intermittent feedback

We consider the two-user Gaussian interference channel with intermittent feedback, illustrated in Fig. 1. We assume Transmitter i (Txi) has a message W_i intended for Receiver i (Rxi), $i = 1, 2$. $W_1 \in \left[2^{NR_1}\right]$ and $W_2 \in \left[2^{NR_2}\right]$ are independent and uniformly distributed, where, for $n \in \mathbb{N}$, $[n] := \{k \in \mathbb{N} : k \leq n\}$. The signal transmitted by Txi at time t is denoted by $X_i[t]$. The channel output received by Rxi at time t, $Y_i[t]$, $i = 1, 2$, is related to the channel inputs at time t, $X_j[t]$, $j = 1, 2$, by the relations

$$Y_1[t] = h_{11}X_1[t] + h_{12}X_2[t] + Z_1[t]$$
$$Y_2[t] = h_{21}X_1[t] + h_{22}X_2[t] + Z_2[t]$$

where $h_{11}, h_{12}, h_{21}, h_{22} \in \mathbb{C}$ are complex channel gains, and $Z_1[t], Z_2[t] \sim \mathcal{CN}(0, 1)$ are circularly symmetric complex Gaussian noise. We assume an average transmit power constraint of P_i at Txi, *i.e.*, $\mathbb{E}[X_i] \leq P_i$, $i = 1, 2$.

We define

$$\mathsf{SNR}_i := |h_{ii}|^2 P_i$$
$$\mathsf{INR}_i := |h_{ij}|^2 P_j$$

for $(i, j) = (1, 2), (2, 1)$, and

$$\alpha_i := \frac{\mathsf{INR}_i}{\mathsf{SNR}_i}$$

for $i = 1, 2$.

The feedback state sequence pair $\underline{S}^N := \left(S_1^N, S_2^N\right)$ have the joint distribution

$$p(S_1^N, S_2^N) = \prod_{t=1}^{N} p(S_1[t], S_2[t]).$$

and marginally, $S_i[t] \sim Bernoulli(p_i)$, for $i = 1, 2$, for all t and N. Note that, for any fixed time slot t, the random variables $S_1[t]$ and $S_2[t]$ are not necessarily independent, that is, the joint distribution $p(S_1[t], S_2[t])$ can be arbitrary. We assume that receivers have access to $\underline{S}$ causally.

At the beginning of time t, Txi observes the channel output received by Rxi at time $t-1$ through an erasure channel, *i.e.*, it receives $\widetilde{Y}_i[t-1] := S_i[t-1]Y_i[t-1]$, for $i = 1, 2$. Note that this is a *passive* feedback model, in that it does not allow the receiver to perform any processing on the channel output; it simply forwards the received signal Y_i at every time slot, which gets erased with probability $1 - p_i$.

We use the notation $X \overset{\mathrm{f}}{=} Y$ to denote that X is a deterministic function of Y[1]. Then $X_i[t] \overset{\mathrm{f}}{=} \left(W_i, S_i^{t-1}, \widetilde{Y}_i^{t-1}\right)$.

We also define

$$V_i := h_{ji}X_i + Z_j,$$
$$\widetilde{V}_i := S_j V_i,$$

for $(i, j) = (1, 2), (2, 1)$.

A vector channel is described by the equations

$$\mathbf{Y}_1[t] = \mathbf{H}_{11}\mathbf{X}_1[t] + \mathbf{H}_{12}\mathbf{X}_2[t] + \mathbf{Z}_1[t]$$
$$\mathbf{Y}_2[t] = \mathbf{H}_{21}\mathbf{X}_1[t] + \mathbf{H}_{22}\mathbf{X}_2[t] + \mathbf{Z}_2[t]$$
$$\widetilde{\mathbf{Y}}_1[t] = S_1[t]\mathbf{Y}_1[t]$$
$$\widetilde{\mathbf{Y}}_2[t] = S_2[t]\mathbf{Y}_2[t]$$

where $\mathbf{H}_{ij} \in \mathbb{C}^{M\times M}$, for $(i, j) = \{1, 2\}^2$, are diagonal matrices whose k'th elements are denoted by $h_{ij}^{(k)}$; $\mathbf{X}_i[t], \mathbf{Y}_i[t] \in \mathbb{C}^M$, $i = 1, 2$, are the channel input and output, respectively, at user i; $\mathbf{Z}_1[t]$ and $\mathbf{Z}_2[t]$ are independent and distributed with $\mathcal{CN}(\mathbf{0}, \mathbf{I})$; and $\widetilde{\mathbf{Y}}_i[t], i = 1, 2$ is the output of the feedback channel of Txi, at time t. Note that at any given time, the same feedback state variable $S_i[t]$ controls the presence of feedback for all sub-channels, *i.e.*, the feedback is present either for all M channels, or for none of them.

A rate pair (R_1, R_2) is said to be achievable if there exists a pair of codebooks $(\mathcal{C}_1, \mathcal{C}_2)$ with rates R_1 and R_2, respectively, and pairs of encoding and decoding functions such that the average probability of error at any decoder goes to 0 as the block length N goes to infinity. The capacity region with feedback probabilities p_1 and p_2, $\mathcal{C}(p_1, p_2)$, is defined as the closure of the set of all achievable rate pairs (R_1, R_2) when $S_1 \sim Bernoulli(p_1)$ and $S_2 \sim Bernoulli(p_2)$.

[1]More formally, for random variables A and B, $A \overset{\mathrm{f}}{=} B$ means that there exists a $\sigma(B)$-measurable function f such that $A = f(B)$ almost surely, where $\sigma(B)$ is the sigma-algebra generated by B.

$$\mathsf{F}_i = \log\left(1 + \frac{\mathsf{SNR}_i}{\mathsf{INR}_j}\right) - C_i - \log 3 \tag{1}$$

$$\mathsf{G}_i = \log\left(1 + \mathsf{SNR}_i + \mathsf{INR}_i\right) - C_i - \log 6 \tag{2}$$

$$\mathsf{H}_i = \log\left(1 + \mathsf{SNR}_i\right) + p_j \log\left(1 + \frac{3\mathsf{INR}_j}{(3 + \mathsf{SNR}_i)(2 + 2D_j)}\right) - C_i - \log 3 \tag{3}$$

$$\mathsf{J}_i = \log\left(1 + \mathsf{INR}_j\right) - C_j - \log 3 \tag{4}$$

$$\mathsf{K}_i = \log\left(1 + \frac{\mathsf{SNR}_i}{\mathsf{INR}_j} + \mathsf{INR}_i\right) + \mathbb{1}_{\{\mathsf{INR}_i \le \mathsf{SNR}_i\}} p_i \log\left(\frac{1 + \frac{\mathsf{INR}_i}{3+2D_i}}{1 + \frac{\mathsf{INR}_i \mathsf{INR}_j}{\mathsf{SNR}_i + 3\mathsf{INR}_j}}\right) - 2C_i - \log 3 \tag{5}$$

$$\mathsf{M}_i = \log\left(1 + \mathsf{SNR}_i + \mathsf{INR}_i\right) - C_i - C_j' - \log 3 \tag{6}$$

III. Main Result

Our main result is summarized in the following theorem.

Theorem 3.1: The capacity region $\mathcal{C}(p_1, p_2)$ of the two-user Gaussian interference channel with intermittent feedback includes $\mathcal{R}(D_1, D_2, p_1, p_2)$, consisting of the rate pairs satisfying

$$R_1 < \min\{\mathsf{G}_1, \mathsf{H}_1, \mathsf{F}_1 + \mathsf{J}_2\} \tag{7}$$

$$R_2 < \min\{\mathsf{G}_2, \mathsf{H}_2, \mathsf{F}_2 + \mathsf{J}_1\} \tag{8}$$

$$R_1 + R_2 < \min\{\mathsf{F}_1 + \mathsf{M}_2, \mathsf{F}_2 + \mathsf{M}_1, \mathsf{K}_1 + \mathsf{K}_2\} \tag{9}$$

$$2R_1 + R_2 < \mathsf{F}_1 + \mathsf{K}_2 + \mathsf{M}_1 \tag{10}$$

$$R_1 + 2R_2 < \mathsf{F}_2 + \mathsf{K}_1 + \mathsf{M}_2 \tag{11}$$

for all distortion constraints $D_1, D_2 > 0$, where F_i, G_i, H_i, J_i, K_i, M_i, $(i,j) = (1,2), (2,1)$, are as defined in (1) – (6), and C_i and C_i' are given by

$$C_i = p_i \log\left(1 + \frac{3}{2D_i}\right) + 2p_j \log\left(1 + \frac{3}{2D_j}\right),$$

$$C_i' = p_i \log\left(1 + \frac{3}{2D_i}\right),$$

for $(i,j) = (1,2), (2,1)$.

Remark 3.1: The rate region given in Theorem 3.1 is consistent with the capacity region for the linear deterministic version of the problem [9].

Remark 3.2: The achievable scheme that will be presented can actually achieve a slightly larger rate region, since we have backed off from the maximum achievable region by a constant amount to make our analysis simpler. In addition, in order to simplify the rate region expressions we have chosen a particular power allocation, which is not necessarily the optimum one.

Corollary 3.1 (Generalized Degrees of Freedom): For symmetric channel parameters ($\mathsf{SNR}_1 = \mathsf{SNR}_2 = \mathsf{SNR}$, $\mathsf{INR}_1 = \mathsf{INR}_2 = \mathsf{INR}$, $p_1 = p_2 = p$), the symmetric generalized degrees of freedom of freedom, defined by

$$d_{\text{sym}} := \lim_{\substack{\mathsf{SNR}\to\infty \\ \mathsf{INR}=\mathsf{SNR}^\alpha}} \frac{C_{\text{sym}}(\mathsf{SNR}, \mathsf{INR}, p)}{\log \mathsf{SNR}},$$

where $C_{\text{sym}}(\mathsf{SNR}, \mathsf{INR}, p) := \sup\{R : (R,R) \in \mathcal{C}(p,p)\}$, is given by

$$d_{\text{sym}} = \begin{cases} \min\{1 - \alpha/2, 1 - (1-p)\alpha\}, & \alpha \le 1/2 \\ \min\{1 - \alpha/2, p + (1-p)\alpha\}, & 1/2 \le \alpha \le 1 \\ \min\{\alpha/2, (1-p) + p\alpha\}, & \alpha \ge 1 \end{cases}$$

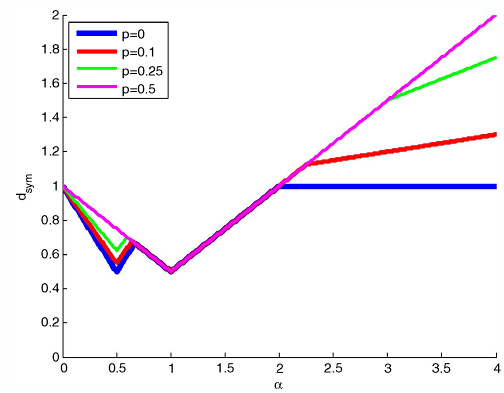

Fig. 2. Generalized degrees of freedom with respect to interference strength $\alpha := \frac{\log \mathsf{INR}}{\log \mathsf{SNR}}$ for symmetric channel parameters. When feedback is available with probability 0.5, the GDoF performance of perfect feedback is achieved.

This result demonstrates that even unreliable feedback can be harnessed to provide *multiplicative gain* in Gaussian interference channels. Fig. 2 shows the symmetric generalized degrees of freedom for various values of feedback probability. As can be seen, as p is gradually increased from 0, we obtain progressively better degrees of freedom curves. It is worth noting that when $p = 0.5$, the V curve obtained by perfect feedback is achieved. Next, we make this observation more precise.

We define sum-capacity as

$$C_{\text{sum}}(p_1, p_2) := \sup\{R_1 + R_2 : (R_1, R_2) \in \mathcal{C}(p_1, p_2)\},$$

and define $\bar{C}^p_{\text{sum}}$ to be the sum rate outer bound for perfect feedback, given in Theorem 3 of [1] as follows.

$$\bar{C}^p_{\text{sum}} = \sup_\rho \min\left\{\log\left(1 + \frac{(1-\rho^2)\mathsf{SNR}_1}{1 + (1-\rho^2)\mathsf{INR}_2}\right) + \log\left(1 + \mathsf{SNR}_2 + \mathsf{INR}_2 + 2\rho\sqrt{\mathsf{SNR}_2 \cdot \mathsf{INR}_2}\right),\right.$$

$$\log\left(1+\frac{(1-\rho^2)\mathsf{SNR}_2}{1+(1-\rho^2)\mathsf{INR}_1}\right) + \log\left(1+\mathsf{SNR}_1+\mathsf{INR}_1+2\rho\sqrt{\mathsf{SNR}_1\cdot\mathsf{INR}_1}\right)\Bigg\}$$

The next corollary shows that when p_1 and p_2 are sufficiently large, the sum-capacity of the perfect feedback channel can be achieved to within a constant gap.

Corollary 3.2: There exists $0 < p^* < 1$ such that[2]

$$\bar{C}^{p}_{\text{sum}} - C_{\text{sum}}(p_1,p_2) = \Omega(1)$$

for all $p_1, p_2 \geq p^*$.

Proof: See [10]. ■

Although the exact threshold p^* does not have a clean closed-form expression, the example of symmetric channel shows that, depending on channel parameters, it can get as low as 0.5. This behavior has a clear intuition behind it. Note that the larger p is, the larger the amount of additional information about the past reception can be obtained through intermittent feedback at the transmitters. If the amount of such information is larger than a threshold, then sending it to the receivers will limit the rate for delivering fresh information. Hence, once this threshold is reached, having more feedback resource is no longer useful. However, this property is not observed for the entire capacity region, since if one of the users transmit at a low rate, then it will have sufficient slackness in rate to forward the entire feedback information.

The next corollary extends the result to vector channels, which are useful for modeling several practical scenarios, such as OFDM, or packet drops over a best-effort network, *e.g.*, WiFi.

Definition 3.1: Minkowski sum of two sets A and B is defined by

$$A \oplus B := \{a+b : a \in A, b \in B\}.$$

Corollary 3.3 (Vector channels): For any vector channel of size M with feedback probabilities p_1 and p_2, the rate region

$$\bigoplus_{k=1}^{M} \mathcal{R}^{(k)}(D_1^{(k)}, D_2^{(k)}, p_1, p_2)$$

is achievable for all sets of distortion constraints $D_1^{(k)}, D_2^{(k)} > 0$, $k \in [M]$, where $\mathcal{R}^{(k)}$ is the achievable rate region for the k'th sub-channel according to Theorem 3.1.

Proof: Decompose any target rate point (R_1, R_2) into M components $(R_1^{(k)}, R_2^{(k)})$ such that $(R_1, R_2) = \sum_{k=1}^{M}(R_1^{(k)}, R_2^{(k)})$, and $(R_1^{(k)}, R_2^{(k)}) \in \mathcal{R}^{(k)}(D_1^{(k)}, D_2^{(k)}, p_1, p_2)$, which is possible if (R_1, R_2) lies in the set given in the corollary, by Definition 3.1. Use sub-channel k independently to send messages with rates $R_1^{(k)}$ and $R_2^{(k)}$. Since $(R_1^{(k)}, R_2^{(k)})$ lies in the achievable region of the sub-channel, the result follows. ■

[2] $\Omega(1)$ is in Bachmann-Landau notation, *i.e.*, in this case, it represents any constant independent of channel parameters.

Remark 3.3: Note that using the sub-channels independently is not necessarily an optimal strategy. It might be possible, in general, to achieve a larger rate region by coding over sub-channels.

IV. Proof of Achievability

A. Overview of the Achievable Strategy

The scheme consists of transmission over B blocks, each of length N. At the beginning of block b, upon reception of feedback, transmitters first remove their own contribution from the feedback signal and obtain a function of the interference and noise realization of block $b-1$. This signal is then quantized and mapped to a new codeword, which will be called the helping information. Finally, new common and private information codewords are superposed to the helping information, and sent to the receiver.

The decoding operation depends on the desired rate point (see Fig. 3). To achieve the rate points for which the common component of the message is large, the receiver simply performs a variation of Han-Kobayashi decoding, *i.e.*, it decodes the intended information jointly with the common part of the interference. Note that this does not make use of the helping information.

To achieve the remaining rate points, the helping information is used. For weak interference, at block b, we assume that the receiver has already decoded the intended common information of block $b-1$. After receiving the transmission of block b, the receiver jointly decode the intended private information and the interference of block $b-1$ jointly with the common information of block b, while using the helping information sent at block b as side information. For strong interference, the roles of intended common information and the interfering common information get switched.

Next, we present a detailed description of the coding scheme and proof of achievability.

B. Codebook Generation

Fix $p(x_{ie})p(x_{ic})p(x_{ip})$ for $i = 1,2$, and $p(u_i|\tilde{v}_j)$ that achieves $\mathbb{E}\left[d(U_i, \widetilde{V}_j)\right] \leq D_i$ for $(i,j) = (1,2),(2,1)$, where $d(\cdot,\cdot)$ is the squared-error distortion measure, given by, $d(x,y) := (x-y)^2$. Generate 2^{Nr_i} quantization codewords U_i^N i.i.d. $\sim p(u_i) = \sum_{\tilde{v}_j} p(u_i|\tilde{v}_j)p(\tilde{v}_j)$, for $(i,j) = (1,2),(2,1)$. For $i = 1,2$, generate 2^{Nr_i} codewords X_{ie}^N i.i.d. $\sim p(x_{ie})$. Further generate, for $i = 1,2$, $2^{NR_{ic}}$ codewords X_{ic}^N i.i.d. $\sim p(x_{ic})$ and $2^{NR_{ip}}$ codewords X_{ip}^N i.i.d. $\sim p(x_{ip})$.

C. Encoding

Encoding is performed over blocks (indexed by b) of length N. See Fig. 4 for a system diagram. At the beginning of block b, Txi receives the punctured feedback signal $\widetilde{Y}_i^N(b-1)$ containing information about the channel output in block $b-1$, given by

$$\widetilde{Y}_i^N(b-1) = S_i^N(h_{ii}X_i^N(b-1) + h_{ij}X_j^N(b-1) + Z_i^N(b-1)),\ (i,j) = (1,2),(2,1)$$

Interference Regime	Operating Point	Decoding Operation
Weak Interference	$R_{1c} < I_{w1}$	Jointly decode $W_{1p}(b-1), W_{2c}(b-1), W_{1c}(b)$, and quantization index $Q_1(b)$
	$R_{1c} \geq I_{w1}$	Jointly decode $W_{1p}(b), W_{1c}(b)$, and $W_{2c}(b)$ (do not use helping information)
Strong Interference	$R_{2c} < I_{s1}$	Jointly decode $W_{1p}(b-1), W_{1c}(b-1), W_{2c}(b)$, and quantization index $Q_2(b)$
	$R_{2c} \geq I_{s1}$	Jointly decode $W_{1p}(b), W_{1c}(b)$, and $W_{2c}(b)$ (do not use helping information)

Fig. 3. A high-level summary of the decoding policy at Rx1.

where multiplication by the vector S_i^N is element-wise. Upon reception of $\widetilde{Y}_i^N$, Txi first subtracts its own contribution $S_i^N h_{ii} X_i^N$ to obtain

$$\widetilde{V}_j^N(b-1) = S_i^N(h_{ij}X_j^N(b-1) + Z_i^N(b-1))$$

for $(i,j) = (1,2),(2,1)$. This signal is then quantized by finding an index $Q_i(b)$ such that

$$\left(\widetilde{V}_j^N(b-1), U_i^N(Q_i(b))\right) \in \mathcal{T}_\epsilon^{(N)},$$

where $\mathcal{T}_\epsilon^{(N)}$ denotes the ϵ-typical set with respect to the distribution $p(\widetilde{v}_j)p(u_i|\widetilde{v}_j)$, and $p(\widetilde{v}_j)$ is induced by the channel and the input distributions. If such an index $Q_i(b)$ has been found, the codeword $X_{ie}^N(Q_i(b))$ that has the same index is chosen to be sent for block b. If there are multiple such indices, the smallest one is chosen. If no such index is found, the quantization index 1 is chosen.

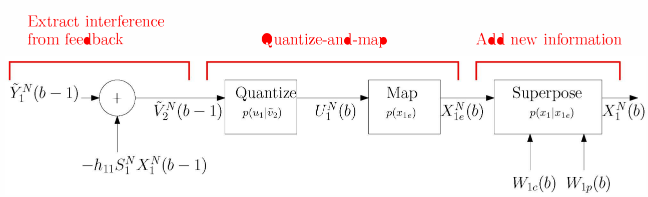

Fig. 4. Encoder diagram at Tx1

Note that if we fix a codebook, the helping information pair $\left(U_i^N(b), X_{ie}^N(b)\right)$ become a function of the interference and noise of the previous block, and the feedback channel realization, that is,

$$\begin{aligned}\left(U_i^N(b), X_{ie}^N(b)\right) &\stackrel{\text{f}}{=} \left(\underline{S}^N(b-1), Y_i^N(b-1), X_i^N(b-1), \mathcal{C}_i\right)\\ &\stackrel{\text{f}}{=} \left(\underline{S}^N(b-1), X_j^N(b-1), Z_i^N(b-1), \mathcal{C}_i\right)\end{aligned}$$

for $(i,j) = (1,2),(2,1)$, and $\mathcal{C}_i$ represents the collection of codebooks at Tx i. Next, the message $W_i(b) \in \left[2^{NR_i}\right]$ to be sent at block b is split into common and private components $(W_{ic}(b), W_{ip}(b)) \in \left[2^{NR_{ic}}\right] \times \left[2^{NR_{ip}}\right]$. Depending on the desired message indices $(W_{ic}(b), W_{ip}(b))$, a common codeword $X_{ic}^N(W_{ic}(b))$, and a private codeword $X_{ip}^N(W_{ip}(b))$ is chosen from the respective codebooks.

Finally, the codeword $X_i^N(b) = X_{ie}^N(b) + X_{ic}^N(b) + X_{ip}^N(b)$ is sent. For convenience, we denote $X_{if}^N(b) := X_{ie}^N(b) + X_{ic}^N(b)$.

D. Decoding

The message indices for common and private messages, and the quantization indices of Txi at block b will be denoted by $m_i(b)$, $n_i(b)$, and $q_i(b)$, respectively. When there are two quantization indices to be decoded, the second one will be denoted with $q_i'(b)$. Rxi receives, during blocks $b-1$ and b,

$$\begin{aligned}Y_i^N(b) &= h_{ii}X_i^N(b) + h_{ij}X_j^N(b) + Z_i^N(b),\\ Y_i^N(b-1) &= h_{ii}X_i^N(b-1) + h_{ij}X_j^N(b-1) + Z_i^N(b-1).\end{aligned}$$

In order to describe the decoding process, we need to introduce some notation. Define the following sequence of sets:

$$\begin{aligned}\mathcal{B}_i^{(N)}((q_j, m_j)(b-1)) := \Big\{ q_i : \big(&\underline{S}^N(b-1),\\ &X_{jf}^N((q_j, m_j)(b-1)), (U_i^N, X_{ie}^N)(q_i(b))\big) \in \mathcal{T}_\epsilon^{(N)} \Big\}.\end{aligned}$$

for $(i,j) = (1,2),(2,1)$. Loosely, $\mathcal{B}_i^{(N)}$ is the set of quantization indices of Txi that are typical with the interference of the previous round. If any of the indices (q_j, m_j) is known, we will suppress the dependence to that index, *e.g.*, if both are known, we simply denote

$$\begin{aligned}\mathcal{B}_i^{(N)}(b) := \Big\{ q_i : \big(&\underline{S}^N(b-1),\\ &X_{jf}^N(b-1), U_i^N(q_i(b)), X_{ie}^N(q_i(b))\big) \in \mathcal{T}_\epsilon^{(N)} \Big\}\end{aligned}$$

where $X_{jf}^N(b-1)$ refers to the codeword corresponding to the known message indices.

We assume that the set $\mathcal{B}_i^{(N)}(b)$ has cardinality $2^{NK_i'(b)}$. Specifically,

$$K_i'(b) = \frac{1}{N}\log\left|\left\{ q_i(b) : \left(\widetilde{V}_j^N(b-1), U_i^N(q_i(b))\right) \in \mathcal{T}_\epsilon^{(N)} \right\}\right|$$

Note that due to random codebook generation, $K_i'(b), i = 1,2$, are random variables. The following lemma shows that $K_i'(b)$ is almost surely bounded by a constant for sufficiently large N.

Lemma 4.1: For any $\epsilon > 0$, there exists a block length N, and a quantization scheme such that $K_i'(b) < C_i' + \delta(\epsilon)$, where

$$C_i' := I(\widetilde{V}_j; U_i) - I(X_{jf}; U_i)$$

for $(i, j) = (1, 2), (2, 1)$, and $\delta(\epsilon)$ is such that $\delta(\epsilon) \to 0$ as $\epsilon \to 0$.

Proof: See [10]. ■

We also define $C_i = C_i' + 2C_j'$, for $(i, j) = (1, 2), (2, 1)$. The reason for this particular definition will become clear in the error analysis. Intuitively, C_i represents the cost associated with sending the helping information to the receiver, in addition to fresh information.

Decoding operation depends on the interference regime and the desired operating point (R_1, R_2). In what follows, for clarity, we will focus only on Rx1. The operations performed at Rx2 are similar.

1) Weak Interference ($\mathsf{INR}_1 < \mathsf{SNR}_1$): Define $I_{w1} := I(X_{1f}; Y_1 | X_{1e}, X_{2e}) - C_1$. If, for the desired operating point, $R_{1c} > I_{w1}$, the helping information is not used, and a slight modification of Han-Kobayashi scheme is employed. Otherwise, the helping information is used to decode the information of block $b-1$. We describe the decoding for the two cases below.

$\mathbf{R_{1c} \geq I_{w1}}$: At block b, we assume that $X_{1e}^N(b)$ and $X_1^N(b-1)$ are known. The decoder attempts to find unique indices $(m_1(b), n_1(b), m_2(b)) \in [2^{NR_{1c}}] \times [2^{NR_{1p}}] \times [2^{NR_{2c}}]$, and some $q_2(b) \in [2^{Nr_2}]$ such that

$$\begin{pmatrix} \underline{S}^N, X_{1f}^N(b-1), X_{1e}^N(b), X_{2e}^N(q_2(b)), X_{1f}^N(m_1(b)), \\ X_1^N(m_1(b), n_1(b)), X_{2f}^N(q_2(b), m_2(b)), Y_1^N(b) \end{pmatrix} \in \mathcal{T}_\epsilon^{(N)} \quad (12)$$

where the known message indices are suppressed. If it can find a unique collection of such indices, it declares them as the decoded message indices $\left(\widehat{W}_{1c}, \widehat{W}_{1p}, \widehat{W}_{2c}\right)$; otherwise it declares an error.

After decoding, given the knowledge of $X_1^N(b)$, Rx1 reconstructs $X_{1e}^N(b+1)$ by imitating the steps taken by Tx1 at the beginning of block $b+1$, thereby maintaining the assumption that $X_{1e}^N(b)$ is known at the beginning of block b.

$\mathbf{R_{1c} < I_{w1}}$: At block b, it is assumed that $X_{1f}^N(b-1)$ and $X_1^N(b-2)$ are known at Rx1.

To decode, Rx1 attempts to find unique indices $(m_1(b), n_1(b-1), m_2(b-1)) \in [2^{NR_{1c}}] \times [2^{NR_{1p}}] \times [2^{NR_{2c}}]$ and some triple $(q_2(b-1), q_2(b), q_1(b)) \in [2^{Nr_2}] \times [2^{Nr_2}] \times [2^{Nr_1}]$ such that

$$\begin{pmatrix} \underline{S}^N(b-1), X_{1f}^N(b-2), X_{1f}^N(b-1), X_1^N(n_1(b-1)), \\ X_{2e}^N(q_2(b-1)), X_{2c}^N(m_2(b-1)), \left(U_1^N, X_{1e}^N\right)(q_1(b)), \\ X_{2e}^N(q_2(b)), X_{1c}^N(m_1(b)), Y_1^N(b-1), Y_1^N(b) \end{pmatrix} \in \mathcal{T}_\epsilon^{(N)} \quad (13)$$

If a unique collection of such indices exists, then these are declared as the decoded message indices $\left(\widehat{W}_{1c}, \widehat{W}_{1p}, \widehat{W}_{2c}\right)$. Otherwise, an error is declared.

In (13), the dependence of $X_1^N(b-1)$ to the indices $q_1(b-1)$ and $m_1(b-1)$ is suppressed, since these indices correspond to messages that have already been decoded.

In words, the decoder jointly decodes the private information and the interference of block $b-1$ jointly with the helping information and common information from block b.

Note that non-unique decoding is performed for $X_{1e}^N(b)$, but we have assumed that $X_{1f}^N(b-1)$ (and thus, $X_{1e}^N(b-1)$) is uniquely known at the beginning of block b. In order to maintain this assumption for the next block, $X_{1e}^N(b)$ is reconstructed at Rx1. To achieve this, given the knowledge of $X_1^N(b-1)$, and the quantization codebook, Rx1 imitates the operations performed by Tx1 at the beginning of block b.

2) Strong Interference ($\mathsf{INR}_1 \geq \mathsf{SNR}_1$): As in the weak interference case, decoding depends on the operating point. For $R_{2c} < I_{s1} := I(X_{2f}; Y_1 | X_{1e}, X_{2e}) - C_1$, helping information is used, otherwise, helping information is not used.

$\mathbf{R_{2c} \geq I_{s1}}$: The operations performed are identical to those for the case of $R_{1c} \geq I_{w1}$ under weak interference, so we omit this case.

$\mathbf{R_{2c} < I_{s1}}$: We assume $X_1^N(b-2)$, $X_{1e}^N(b-1)$, and $X_{2c}^N(b-1)$ are known at Rx1 at block b.

To decode, Rx1 attempts to find unique indices $(m_1(b-1), n_1(b-1), m_2(b)) \in [2^{NR_{1c}}] \times [2^{NR_{1p}}] \times [2^{NR_{2c}}]$ and some $(q_2(b-1), q_2(b), q_1(b)) \in [2^{Nr_2}] \times [2^{Nr_2}] \times [2^{Nr_1}]$ such that

$$\begin{pmatrix} \underline{S}^N(b-1), X_{1f}^N(b-2), X_{1e}^N(b-1), X_{2e}^N(q_2(b-1)), \\ X_{1c}^N(m_1(b-1)), X_{1p}^N(n_1(b-1)), X_{2c}^N(m_2(b)), \\ (U_2^N, X_{2e}^N)(q_2(b)), X_{1e}^N(q_1(b)), Y_1^N(b-1), Y_1^N(b) \end{pmatrix} \in \mathcal{T}_\epsilon^{(N)} \quad (14)$$

If a unique collection of such indices exists, they are declared as the decoded message indices $\left(\widehat{W}_{1c}, \widehat{W}_{1p}, \widehat{W}_{2c}\right)$. Otherwise, an error is declared. Using the information of $X_1^N(b-1)$, Rx1 can now uniquely reconstruct $X_{1e}^N(b-1)$ by following the steps taken by Tx1 at the beginning of block b.

E. Error Analysis

We focus only on Rx1, and an arbitrary block b for simplicity. Without loss of generality, we consider the error events occurring at Tx1 and Rx1. All arguments here will be applicable to the other Tx-Rx pair. We define the following decoding error events at Rx1:

$$D_{FB,w}(b) = \Big\{\widehat{W}_{1c}(b) = W_{1c}(b), \widehat{W}_{1p}(b-1) = W_{1p}(b-1), \widehat{W}_{2c}(b-1) = W_{2c}(b-1)\Big\}^c$$

$$D_{FB,s}(b) = \Big\{\widehat{W}_{1c}(b-1) = W_{1c}(b-1),$$

$$\widehat{W}_{1p}(b-1) = W_{1p}(b-1), \widehat{W}_{2c}(b) = W_{2c}(b)\Big\}^c$$

$$D_{NFB}(b) = \left\{\widehat{W}_1(b) = W_1(b), \widehat{W}_{2c}(b) = W_{2c}(b)\right\}^c$$

Now we analyze the weak and strong interference regimes separately.

1) Weak Interference: The following lemmas characterize the rate constraints for reliable communication with Rx1 for feedback and non-feedback strategies, respectively, under weak interference.

Lemma 4.2: For $\alpha_1 < 1$, $\mathbb{P}(D_{FB,w}(b)) \to 0$ as $N \to \infty$ if

$$R_{1c} < I(X_{1f}; Y_1 | X_{1e}, X_{2e}) - C_1 \quad (15)$$

$$R_{1p} < I(X_1; Y_1 | X_{1f}, X_{2f}) - C_1 \quad (16)$$

$$R_{2c} < I(X_{2f}; Y_1 | X_{2e}, X_1) - C_1 \quad (17)$$

$$R_{1p} + R_{2c} < \min\Big\{I(X_1, X_{2f}; Y_1, U_1 | X_{1f}, X_{2e}) \quad (18)$$

$$I(X_1, X_{2f}; Y_1 | X_{1c}, X_{2e})\Big\} - 2C_1 \quad (19)$$

$$R_1 + R_{2c} < I(X_1, X_{2f}; Y_1 | X_{1e}, X_{2e}) - C_1 - C_2' \quad (20)$$

Proof: See [10]. ■

Lemma 4.3: For $\alpha_1 < 1$, $\mathbb{P}(D_{NFB}(b)) \to 0$ as $N \to \infty$ if

$$R_{1c} > I(X_{1f}; Y_1 | X_{1e}, X_{2e}) - C_1 \quad (21)$$

$$R_{1p} < I(X_1; Y_1 | X_{1f}, X_{2f}) - C_1 \quad (22)$$

$$R_{2c} < I(X_{2f}; Y_1 | X_{2e}, X_1) - C_1 \quad (23)$$

$$R_1 < I(X_1; Y_1 | X_{2f}, X_{1e}) - C_1 \quad (24)$$

$$R_1 + R_{2c} < I(X_1, X_{2f}; Y_1 | X_{1e}, X_{2e}) - C_1 - C_2' \quad (25)$$

Proof: See [10]. ■

Recall that feedback mode is used at Rx1 only if (15) is satisfied; otherwise Han-Kobayashi decoding is performed. If we define $\underline{R} := (R_{1c}, R_{2c}, R_{1p})$, and

$$\mathcal{R}^w_{FB} := \{\underline{R} : \text{(16)-(20) is satisfied}\},$$

$$\mathcal{R}^w_{NFB} := \{\underline{R} : \text{(22)-(25) is satisfied}\},$$

$$\mathcal{R}^w_d := \{\underline{R} : \text{(15) is satisfied}\},$$

then the set of rate points $\mathcal{R}^w$ that ensure decodability at Rx1 under weak interference contains

$$\begin{aligned}\mathcal{R}^w &= (\mathcal{R}^w_{FB} \cap \mathcal{R}^w_d) \cup (\mathcal{R}^w_{NFB} \cap \mathcal{R}^{w,c}_d) \\ &\supseteq (\mathcal{R}^w_{NFB} \cap \mathcal{R}^w_{FB} \cap \mathcal{R}^w_d) \cup (\mathcal{R}^w_{NFB} \cap \mathcal{R}^w_{FB} \cap \mathcal{R}^{w,c}_d) \\ &= \mathcal{R}^w_{NFB} \cap \mathcal{R}^w_{FB}\end{aligned}$$

where $\mathcal{R}^{w,c}_d$ is the complement of the set $\mathcal{R}^w_d$. Therefore, the rate constraints for decodability at Rx1 for the described strategy for weak interference are given by (16)-(20) and (22)-(25), for all joint distributions $\prod_{i=1}^2 p(x_{ie})p(x_{ic})p(x_{ip})$ and $p(u_i|\tilde{v}_j)$, $(i,j) = (1,2),(2,1)$, consistent with the distortion constraints.

2) Strong Interference: The following lemmas give the rate constraints for the feedback and non-feedback modes under strong interference at Rxi.

Lemma 4.4: For $\alpha_1 \geq 1$, $\mathbb{P}(D_{FB,s}(b)) \to 0$ as $N \to \infty$ if

$$R_{2c} < I(X_{2f}; Y_1 | X_{1e}, X_{2e}) - C_1 \quad (31)$$

$$R_{1p} < I(X_1; Y_1 | X_{1f}, X_{2f}) - C_1 \quad (32)$$

$$R_1 < \min\Big\{I(X_1; Y_1, U_2 | X_{1e}, X_{2f}), \quad (33)$$

$$I(X_1, X_{2e}; Y_1 | X_{1e}, X_{2c})\Big\} - C_1 \quad (34)$$

$$R_1 + R_{2c} < I(X_1, X_{2f}; Y_1 | X_{1e}, X_{2e}) - C_1 - C_2' \quad (35)$$

Proof: See [10]. ■

Lemma 4.5: For $\alpha_1 \geq 1$, $\mathbb{P}(D_{NFB}(b)) \to 0$ as $N \to \infty$ if

$$R_{2c} > I(X_{2f}; Y_1 | X_{1e}, X_{2e}) - C_1 \quad (36)$$

$$R_{1p} < I(X_1; Y_1 | X_{1f}, X_{2f}) - C_1 \quad (37)$$

$$R_{1p} + R_{2c} < I(X_1, X_{2f}; Y_1 | X_{1f}, X_{2e}) - 2C_1 \quad (38)$$

$$R_1 + R_{2c} < I(X_1, X_{2f}; Y_1 | X_{1e}, X_{2e}) - C_1 - C_2' \quad (39)$$

Proof: See [10]. ■

One can perform the same line of arguments as in the case of weak interference to show that the rate constraints for decodability at Rx1 for strong interference are given by (32)-(35) and (37)-(39), for all joint distributions $\prod_{i=1}^2 p(x_{ie})p(x_{ic})p(x_{ip})$ and $p(u_i|\tilde{v}_j)$, $(i,j) = (1,2),(2,1)$, consistent with the distortion constraints.

F. Rate Region Evaluation

Now we evaluate the rate constraints obtained in the previous section, and obtain the final achievable rate region. Assuming available power P_i at Txi, we assign the following input distributions, for $(i,j) = (1,2),(2,1)$:

$$\begin{aligned}X_{ie} &\sim \mathcal{CN}(0, P_{ie}) \\ X_{ic} &\sim \mathcal{CN}(0, P_{ic}) \\ X_{ip} &\sim \mathcal{CN}(0, P_{ip}) \\ U_i|\widetilde{V}_j &\sim \mathcal{CN}(\widetilde{V}_j, D_i)\end{aligned}$$

where $D_i > 0$ are the distortion parameters, and $P_i = P_{ie} + P_{ic} + P_{ip}$ for $i = 1,2$. Let us define $\underline{P} := (P_{1e}, P_{1c}, P_{1p}, P_{2e}, P_{2c}, P_{2p})$ for convenience. Using these input distributions, it is easy to evaluate the mutual information terms obtained in the previous sections, and obtain the rate constraints given in (26)–(30), for decodability at Rxi.

for any non-zero power allocation (P_{ie}, P_{ic}, P_{ip}) such that $P_{ie} + P_{ic} + P_{ip} = P_i$ and for $(i,j) = (1,2),(2,1)$, where

$$\begin{aligned}C_i &= p_i \log\left(1 + \frac{1 + |h_{ij}|^2 P_{jp}}{D_i}\right) \\ &\quad + 2p_j \log\left(1 + \frac{1 + |h_{ij}|^2 P_{jp}}{D_j}\right), \\ C_j' &= p_j \log\left(1 + \frac{1 + |h_{ij}|^2 P_{jp}}{D_j}\right)\end{aligned}$$

and the indicator functions are used to unify the rate constraints obtained for weak and strong interference. In terms of evaluation, the only non-trivial bounds here are (28) and (29), whose evaluation is given in [10].

$$R_{ip} < \mathsf{A}_i := \log\left(1 + \frac{|h_{ii}|^2 P_{ip}}{1 + |h_{ij}|^2 P_{jp}}\right) - C_i \tag{26}$$

$$R_{jc} < \mathsf{B}_i := \log\left(1 + \frac{|h_{ij}|^2 P_{jc}}{1 + |h_{ij}|^2 P_{jp}}\right) - C_i \tag{27}$$

$$R_i < \mathsf{C}_i := \min\Bigg\{\log\left(1 + \frac{|h_{ii}|^2 (P_{ic} + P_{ip}) + |h_{ij}|^2 P_{je}}{1 + |h_{ij}|^2 P_{jp}}\right), \log\left(1 + \frac{|h_{ii}|^2 (P_{ic} + P_{ip})}{1 + |h_{ij}|^2 P_{jp}}\right) + \mathbb{1}_{\{|h_{ii}|<|h_{ij}|\}} p_j \log\left(1 + \frac{\frac{|h_{ji}|^2 (P_{ic}+P_{ip})}{1+D_j}}{1 + \frac{|h_{ii}|^2 (P_{ic}+P_{ip})}{1+|h_{ij}|^2 P_{jp}}}\right)\Bigg\} - C_i \tag{28}$$

$$R_{ip} + R_{jc} < \mathsf{D}_i := \min\Bigg\{\log\left(1 + \frac{|h_{ii}|^2 (P_{ie} + P_{ip}) + |h_{ij}|^2 P_{jc}}{1 + |h_{ij}|^2 P_{jp}}\right), \log\left(1 + \frac{|h_{ii}|^2 P_{ip} + |h_{ij}|^2 P_{jc}}{1 + |h_{ij}|^2 P_{jp}}\right) + \mathbb{1}_{\{|h_{ii}|\geq|h_{ij}|\}} p_i \log\left(\frac{1 + \frac{|h_{ij}|^2 P_{jc}}{1+|h_{ij}|^2 P_{jp} + \min(D_i, |h_{ii}|^2 P_{ip})}}{1 + \frac{|h_{ij}|^2 P_{jc}}{1+|h_{ij}|^2 P_{jp}+|h_{ii}|^2 P_{ip}}}\right)\Bigg\} - 2C_i \tag{29}$$

$$R_i + R_{jc} < \mathsf{E}_i := \log\left(1 + \frac{|h_{ii}|^2 (P_{ic} + P_{ip}) + |h_{ij}|^2 P_{jc}}{1 + |h_{ij}|^2 P_{jp}}\right) - C_i - C'_j \tag{30}$$

Applying Fourier-Motzkin elimination, we obtain the following achievable rate region

$$R_1 < \min\{\mathsf{C}_1, \mathsf{A}_1 + \mathsf{B}_2\} \tag{40}$$

$$R_2 < \min\{\mathsf{C}_2, \mathsf{A}_2 + \mathsf{B}_1\} \tag{41}$$

$$R_1 + R_2 < \min\{\mathsf{A}_1 + \mathsf{E}_2, \mathsf{A}_2 + \mathsf{E}_1, \mathsf{D}_1 + \mathsf{D}_2, \mathsf{A}_1 + \mathsf{B}_1 + \mathsf{D}_2, \mathsf{A}_2 + \mathsf{B}_2 + \mathsf{D}_1\} \tag{42}$$

$$2R_1 + R_2 < \mathsf{A}_1 + \mathsf{D}_2 + \mathsf{E}_1 \tag{43}$$

$$R_1 + 2R_2 < \mathsf{A}_2 + \mathsf{D}_1 + \mathsf{E}_2 \tag{44}$$

for any power allocation $\underline{P}$ consistent with the power constraints P_1 and P_2. The following lemma shows that this rate region can be simplified.

Lemma 4.6: In the rate region given by (40)-(44), the following bounds are redundant.

$$R_1 + R_2 < \min\{\mathsf{A}_1 + \mathsf{B}_1 + \mathsf{D}_2, \mathsf{A}_2 + \mathsf{B}_2 + \mathsf{D}_1\}$$

Proof: See [10]. ■

G. Power Allocation

As a final step, we use a power allocation that is a variation of the one used in [11].

$$P_{ip} = \frac{1}{2}\min\left(\frac{1}{|h_{ji}|^2 P_i}, 1\right) P_i,$$

$$P_{ic} = \frac{1}{2}(1 - P_{ip}) P_i$$

$$P_{ie} = \frac{1}{2} P_i.$$

for $i = 1, 2$.

Using this power allocation, and using the definitions of SNR_i and INR_i, it can be seen that the rate region described by (40) – (44) (excluding the bounds shown to be redundant in Lemma 4.6) contains the rate region given in 3.1.

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