On Uplink/Downlink Full-Duplex Networks

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***Abstract*—Recent results in wireless full-duplex promise rate gains over the half-duplex counterpart when two nodes exchange messages with each other. However, when multiple full-duplex nodes operate simultaneously, the resulting network has increased internode interference compared to the half-duplex counterpart. The increased internode interference can potentially limit the rate gain achievable due to introduction of full-duplex capability. In this paper, we present new interference management strategies tha handle internode interference for full-duplex enabled network and achieve rate gains over its half-duplex counterpart.**

I. Introduction

The feasibility of wireless multiple input multiple output (MIMO) full-duplex [1, 2] has opened up a wide range of possibilities of applying full-duplex to multi-user communication. An interesting application of MIMO full-duplex is to enable simultaneous uplink and downlink in a cellular network. However, simultaneous operation of uplink and downlink in the same band by using full-duplex enabled nodes introduces internode interference in the network. To appreciate internode interference in a full-duplex network, consider a network consisting of a base-station (BS) and two mobile users, all of which are capable of operating in full-duplex (see Fig. 1b). Being full-duplex capable, both mobile users can manage self-interference and therefore transmit in uplink and receive in downlink. However, since both users transmit and receive in the same band at the same time, the uplink transmission from mobile user 1 will interfere with downlink reception of mobile user 2 (and vice versa). Note that, internode interference present in a full-duplex enabled network is absent in its half-duplex counterpart. In presence of such internode interference, it is not immediately clear whether full-duplex when applied to multi-user settings has any degree-of-freedom (DoF) gain over half-duplex. The focus of this paper is to present internode interference management strategies for full-duplex uplink/downlink channel which result in higher DoF than the half-duplex counterpart.

Strategies to manage internode interference, arising due to full-duplex operation of nodes in a multi-user network, have been studied in [3, 4]. In [4], we proposed an interference alignment based strategy for the specific case of uplink/downlink channel where an L-antenna full-duplex base-station communicates with K single-antenna full-duplex mobile users (see Fig. 1a). We showed that with the help of ergodic interference alignment, a full-duplex enabled network achieves larger DoF-region than its half-duplex counterpart, as long as the number of mobile users exceeds the number of antennas at the base-station. A direct consequence of this result is that when the number of antennas at the base-station are same as the number of mobile users, half-duplex time-sharing between uplink and downlink is DoF-optimal. However, it is to be noted that our DoF analysis assumes that the strength of uplink, downlink and internode interference channel is same, which presents us with an incomplete picture of the gains possible from full-duplex network. In general, the strength of uplink/downlink channel can be significantly different from the strength of the internode interference channel and a more generalized analysis than DoF is required to capture its impact.

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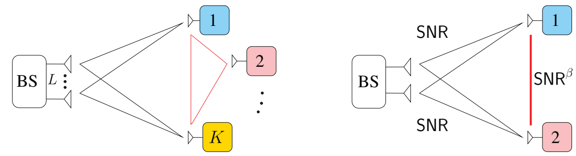


(a) (L, K) network (b) $(2, 2)$ network

Fig. 1: Full-duplex uplink/downlink network

In this paper, we generalize the DoF analysis for a full-duplex network with 2 antennas at the base-station and 2 mobile users, by studying its capacity scaling when the relative strength of the interference channel is different from the uplink/downlink channel (see Fig. 1b). We show that for such a network full-duplex has higher generalized DoF than half-duplex whenever the strength of internode interference is different from the uplink/downlink channel.

The rest of the paper is organized as follows. In Section II, we describe the system model. In Section III, we present the main results discussed in this paper. In Section IV, we review the interference alignment strategy we proposed in [4], and remark on its pros and cons. In Section V, we study generalized DoF for a network with 2 antenna BS and 2 single-antenna mobile users by showing optimal achievable schemes and tight outer-bounds.

II. System Model

Network: We consider a network consisting of one L-antenna full-duplex base-station and K single-antenna full-duplex mobile users (see Fig. 1a). We call such a network an (L, K) *full-duplex network*. Let the uplink, downlink channel between the k^{th} mobile user and BS be denoted by $\mathbf{h}_{\text{UL},k}[i]$, $\mathbf{h}_{\text{DL},k}[i]$ respectively. Let the internode interference channel from all mobile users to the k^{th} mobile user be denoted by $\mathbf{h}_{\text{I},k}[i]$. Then, the input-output relationship in the (L, K) full-duplex network at time instant i is given by

$$\mathbf{y}_{\text{BS}}[i] = \sum_{k=1}^{K} \mathbf{h}_{\text{UL},k}[i]u_k[i] + \mathbf{z}_{\text{BS}}[i] \quad (1a)$$

$$y_k[i] = \mathbf{h}^T_{\text{DL},k}[i]\mathbf{d}[i] + \mathbf{h}^T_{\text{I},k}[i]\mathbf{u}[i] + z_k[i], \quad (1b)$$

where $\mathbf{d}[i]$ and $u_k[i]$ are signals transmitted by the base-station in downlink and k^{th} mobile user in uplink respectively, $\mathbf{u}[i] \equiv [u_1[i], \ldots, u_K[i]]^T$. We assume that the full-duplex capability of BS and mobile users is such that it allows them to completely eliminate all the self-interference. Hence, the k^{th} entry of $\mathbf{h}_{\text{I},k}[i]$ is always zero. Thermal noise at the receivers of BS and mobile users is denoted by $\mathbf{z}_{\text{BS}}[i]$, $z_k[i]$ respectively, whose individual entries are drawn from $\mathcal{N}(0, \sigma_z^2)$. The base-station as well as the mobile users have a transmit power constraint of P, i.e.,

$$\mathbb{E}(|\mathbf{d}[i]|^2) \leq P; \; \mathbb{E}(|u_k[i]|^2) \leq P. \tag{2}$$

Channel Model and Channel Knowledge: We consider a fast fading channel model, where the phase of the uplink/downlink and internode interference channel independently fade at every time instant i. For the purpose of simplifying the analysis, the amplitude of the channel is assumed to be fixed. For an arbitrary (L, K) full-duplex network, we consider the set of channel states such that

$$\frac{||\mathbf{h}_{\text{UL},k}[i]||^2 P}{\sigma_z^2} = \frac{||\mathbf{h}_{\text{DL},k}[i]||^2 P}{\sigma_z^2} = \frac{||\mathbf{h}_{\text{I},k}[i]||^2 P}{\sigma_z^2} = \mathsf{SNR}. \tag{3}$$

Equation (3) indicates that the strengths of uplink/downlink and internode interference channels are identical. We also assume that the parameter SNR is known to all the nodes in the network. For the specific case of $(2, 2)$ full-duplex network, we generalize our analysis to the set of channel states where

$$\frac{||\mathbf{h}_{\text{I},k}[i]||^2 P}{\sigma_z^2} = \mathsf{SNR}^{\beta}, \tag{4}$$

while the strengths of uplink and downlink channels obeys (3). In (4), β captures the strength of the internode interference channel relative to uplink/downlink channel. The assumption regarding channel knowledge is that all nodes in the network are assumed to have instantaneous access to global channel state information at the transmitters and receivers.

Encoding and Decoding: The message $W_{k\to\text{BS}}$ to be transmitted from k^{th} mobile user to BS in uplink is drawn from the message set $\mathcal{W}_{\text{k}\to\text{BS}}$, and the message tuple $(W_{\text{BS}\to 1}, \ldots W_{\text{BS}\to K})$, to be transmitted from BS in downlink, is drawn from a message set $\mathcal{W}_{\text{BS}\to 1} \times \ldots \times \mathcal{W}_{\text{BS}\to K}$. The encoding at BS and mobile users is given by

$$\begin{aligned} \mathbf{d}[i] &= f_{\text{BS}i}(W_{\text{BS}\to 1}, \ldots, W_{\text{BS}\to K}, \mathbf{y}_{\text{BS}}^{i-1}) \\ u_k[i] &= f_{ki}(W_{k\to\text{BS}}, y_k^{i-1}). \end{aligned} \tag{5}$$

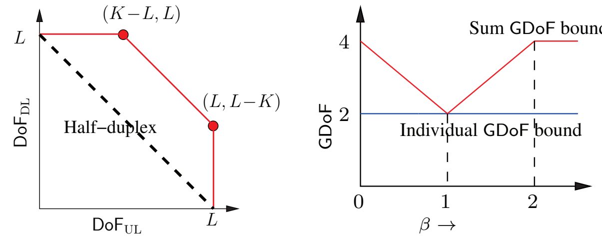
Although, not explicitly shown, the encoding at all the nodes implicitly depends on the channel knowledge till time i.

Assuming a blocklength of N, the decoding at the base-station and the mobile nodes is given by

$$\begin{aligned} \left(\widehat{W}_{1\to\text{BS}}, \times \ldots, \times, \widehat{W}_{K\to\text{BS}}\right) &= g_{\text{BS}}(\mathbf{y}_{\text{BS}}^N) \\ \widehat{W}_{\text{BS}\to k} &= g_k(y_k^N), \end{aligned} \tag{6}$$

where $\widehat{W}_{k\to\text{BS}} \in \mathcal{W}_{k\to\text{BS}}$, $\widehat{W}_{\text{BS}\to k} \in \mathcal{W}_{\text{BS}\to k}$.

Performance Metric: The probability of error is defined as the probability that at least one of the messages is not decoded correctly, that is



(a) DoF-region of an (L, K) full-duplex network when $L < K < 2L$

(b) Bounds on GDoF as a function of β for a $(2, 2)$ full-duplex network

Fig. 2: Illustrations of results in Theorem 1 and 2.

$$p_{\text{error}} = p\left(\bigcup_{\mathsf{X}\in\{k\to\text{BS},\text{BS}\to k\},k=1,\ldots,K} (\widehat{W}_{\mathsf{X}} \neq W_{\mathsf{X}})\right). \tag{7}$$

The rate tuple in uplink $(R_{1\to\text{BS}}, \ldots, R_{K\to\text{BS}})$ and in downlink $(R_{\text{BS}\to 1}, \ldots, R_{\text{BS}\to K})$ are achievable if there exist encoding and decoding functions such that $p_{\text{error}} \to 0$ as $N \to \infty$. For the (L, K) full-duplex network, where the uplink/downlink channel and internode interference channel have the same strength, the DoF in uplink and downlink are defined as

$$\mathsf{DoF}_{\text{UL}} = \lim_{\mathsf{SNR}\to\infty} \frac{\sum_{k=1}^{K} R_{k\to\text{BS}}}{\log \mathsf{SNR}}; \; \mathsf{DoF}_{\text{DL}} = \lim_{\mathsf{SNR}\to\infty} \frac{\sum_{k=1}^{K} R_{\text{BS}\to k}}{\log \mathsf{SNR}}. \tag{8}$$

The DoF-region is set of all achievable DoF-pairs $(\mathsf{DoF}_{\text{UL}}, \mathsf{DoF}_{\text{DL}})$. For the specific case of $(2, 2)$ full-duplex network, we study the generalization of DoF by parametrizing the internode interference channel strength using β. The generalization of DoF parametrized by β is called GDoF.

III. Main-Results

The following theorem describes the DoF-region of an (L, K) full-duplex network

Theorem 1 ([4]): For an (L, K) full-duplex network, the DoF-region is given by

$$\mathsf{DoF}_{\text{UL}} \leq \min(L, K) \tag{9a}$$

$$\mathsf{DoF}_{\text{DL}} \leq \min(L, K) \tag{9b}$$

$$\mathsf{DoF}_{\text{UL}} + \mathsf{DoF}_{\text{DL}} \leq \min(2L, K) \tag{9c}$$

Even if the nodes in the network were only half-duplex capable, the upper bound of DoF in uplink and downlink shown in (9a) and (9b) is achievable (via receive zero-forcing in uplink and transmit zero-forcing in downlink). We generalize Theorem 1 for a $(2, 2)$ full-duplex network by investigating the DoF as a function of internode interference channel strength.

Theorem 2: For a $(2, 2)$ full-duplex network GDoF-region is given by

$$\mathsf{GDoF}_{\text{UL}} \leq 2 \tag{10a}$$

$$\mathsf{GDoF}_{\text{DL}} \leq 2 \tag{10b}$$

$$\mathsf{GDoF}_{\text{UL}} + \mathsf{GDoF}_{\text{DL}} \leq \begin{cases} 2(2-\beta) & \text{if} \quad 0 \leq \beta < 1 \\ \min(2\beta, 4) & \text{if} \quad \beta \geq 1 \end{cases} \tag{10c}$$

IV. THE (L, K) FULL-DUPLEX NETWORK

From Theorem 1, we know that the DoF-region of full-duplex is same as that of half-duplex as long as the number of antennas at BS $\geq$ number of single antenna mobile users. For such networks, internode interference avoidance, achieved via half-duplex time-sharing of uplink/downlink is DoF-optimal strategy. However, as the number of users exceeds the number of antennas at the base-station, a network with full-duplex capable nodes has a higher DoF-region than their half-duplex counterpart and interference avoidance becomes a sub-optimal strategy. In [4], we proposed an internode interference alignment based strategy and showed that it was in fact DoF-optimal. Here, we illustrate the strategy through an example of a network composed of a 2-antenna BS and 3 mobile users.

Internode interference management: From Theorem 1, we know that the optimal sum-DoF for a $(2, 3)$ full-duplex network is 3 and a DoF-pair that achieves the optimal sum-DoF is $(3/2, 3/2)$. The DoF-pair $(3/2, 3/2)$ can be achieved if over two time-slots, 3 independent messages can be delivered both in uplink as well as downlink. In the following discussion, we show how we can enable effective interference management strategy through encoding over two appropriately chosen time-slots. Now, from the perspective of user 1,

$$\text{interference at time i} = h_{\mathrm{I},21}[i]u_2[i] + h_{\mathrm{I},31}[i]u_3[i], \quad (11)$$

where $h_{\mathrm{I},mn}[i]$ is internode interference channel from m^{th} mobile user to the n^{th} mobile user. Suppose there exists another time-slot, say time j, where the interference at user 1 is same as the interference at time i. Then, user 1 can subtract the received signal at time j, $y_1[j]$, from the received signal at time i, $y_1[i]$, to negate all the interference. The first key step in our achievable scheme is to enable such possibilities. The interference at time i will be the same as interference interference at time j, if the internode interference channel at both times is identical and the uplink messages transmitted by interfering users in both times is also identical, i.e.,

$$\begin{aligned} h_{\mathrm{I},21}[i] &= h_{\mathrm{I},21}[j]; \qquad & h_{\mathrm{I},31}[i] &= h_{\mathrm{I},31}[j] \\ u_2[i] &= u_2[j]; & u_3[i] &= u_3[j], \end{aligned} \quad (12)$$

In fact, if for every pair of mobile users, the internode interference channel at time i is identical to the internode interference channel at time j, and each mobile user transmits identical messages at time i and j, then the internode interference at all the users in both time slots will be identical. Thus, an important step in managing interference is to pair up two time slots i,j such that the internode interference channel in both time-slots is the same, and ensure that the mobile users repeat their uplink transmission in those two time-slots.

Decoding in uplink: Repetition of uplink symbols in two time-slots is also useful for decoding them at the uplink receiver, i.e., at the BS. To decode three independent messages from three users, BS needs at least three linearly independent combinations of the messages. Since the base-station has only 2 receive antennas, in every time slot it has access to at most 2 linear combinations. When the uplink symbols transmitted at time i are repeated at time j, the base-station receives a total 4 linear combinations of 3 independent messages. As long as the combined uplink channel at time i and time j given by

$$\mathbf{H}_{\mathrm{UL}} = \begin{bmatrix} \mathbf{h}_{\mathrm{UL},1}[i] & \mathbf{h}_{\mathrm{UL},2}[i] & \mathbf{h}_{\mathrm{UL},3}[i] \\ \mathbf{h}_{\mathrm{UL},1}[j] & \mathbf{h}_{\mathrm{UL},2}[j] & \mathbf{h}_{\mathrm{UL},3}[j] \end{bmatrix} \quad (13)$$

is full-rank, the base-station can decode all the 3 uplink messages. Thus, in addition to the internode interference channel being the same at time i and j, the uplink channels must satisfy rank$(\mathbf{H}_{\mathrm{UL}})=3$, to ensure an uplink DoF of $3/2$.

Encoding/decoding in downlink: The purpose of negating interference at time-slot i from interference at time j is to assist downlink receivers in decoding their intended messages. One strategy that BS can employ for downlink transmission is to use transmit zero-forcing in time slot i and remain silent in time-slot j. Since the base-station has only two antennas, in one time slot it can transmit via zero-forcing to at most two mobile users, say user 1 and 2. At users 1 and 2, subtracting the received signal at time slot j from the received signal at time slot i will eliminate the interference and let them recover their intended messages. Such a strategy implies 2 independent messages can be transmitted in downlink in two time slots resulting in a downlink DoF of $2/2=1$.

The downlink strategy described above attains better sum-DoF than the half-duplex counterpart, however it is strictly sub-optimal. The sub-optimality of the downlink strategy is due to the BS not using both time slots to transmit. Suppose that, the base-station did transmit in both the time-slots, then at user 1 the output after negating the interference will be (ignoring noise)

$$\begin{aligned} y_1[i] - y_1[j] &= \mathbf{h}^T_{\mathrm{DL},1}[i]\mathbf{d}[i] - \mathbf{h}^T_{\mathrm{DL},1}[j]\mathbf{d}[j] \\ &= \begin{bmatrix} \mathbf{h}^T_{\mathrm{DL},1}[i] & \mathbf{h}^T_{\mathrm{DL},1}[j] \end{bmatrix} \begin{bmatrix} \mathbf{d}[i] \\ -\mathbf{d}[j] \end{bmatrix} \end{aligned} \quad (14)$$

Similar outputs can also be obtained at the other two users. The downlink channel, for all three mobile users, combined over the two time slots can be written as

$$\mathbf{H}_{\mathrm{DL}} = \begin{bmatrix} \mathbf{h}^T_{\mathrm{DL},1}[i] & \mathbf{h}^T_{\mathrm{DL},1}[j] \\ \mathbf{h}^T_{\mathrm{DL},2}[i] & \mathbf{h}^T_{\mathrm{DL},2}[j] \\ \mathbf{h}^T_{\mathrm{DL},3}[i] & \mathbf{h}^T_{\mathrm{DL},3}[j] \end{bmatrix}. \quad (15)$$

Now, suppose that the three mobile users intend to receive independent codewords w_1, w_2, w_3 respectively over two time slots, then the base-station can design

$$\begin{bmatrix} \mathbf{d}[i] \\ -\mathbf{d}[j] \end{bmatrix} = \mathbf{H}^{-1}_{\mathrm{DL}} \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix}, \quad (16)$$

where $\mathbf{H}^{-1}_{\mathrm{DL}}$ is the pseudo-inverse of $\mathbf{H}_{\mathrm{DL}}$. It is easy to see that design of downlink transmit vectors as in (16) implies that the k^{th} mobile user can recover w_k by performing $y_k[i] - y_k[j]$. One challenge in employing the design in (16) is the following. In order to design $\mathbf{d}[i]$ at time i, the downlink channels at time j, i.e., $\mathbf{h}^T_{\mathrm{DL},1}[j], \mathbf{h}^T_{\mathrm{DL},2}[j], \mathbf{h}^T_{\mathrm{DL},3}[j]$ need to be known, which is effectively acausal knowledge of downlink channel of time j. Acausal knowledge of channels is not feasible. However, the base-station can design the downlink transmit vectors assuming

$$\mathbf{H}_{\rm DL}=\begin{bmatrix}\mathbf{h}^T_{{\rm DL},1}[i] & \mathbf{h}^T_{{\rm DL},1}[i]\\ \mathbf{h}^T_{{\rm DL},2}[i] & \omega\mathbf{h}^T_{{\rm DL},2}[i]\\ \mathbf{h}^T_{{\rm DL},3}[i] & \omega^2\mathbf{h}^T_{{\rm DL},3}[i]\end{bmatrix}. \tag{17}$$

BS can transmit $\mathbf{d}[i]$ at time i assuming (17) and then wait for the right time index j when $\mathbf{h}^T_{{\rm DL},k}[j]=\omega^{k-1}\mathbf{h}^T_{{\rm DL},k}[i]$, $k=1,2,3$, to transmit the downlink vector $\mathbf{d}[j]$. Note that, by ensuring to pick a time index j where the downlink channel is deterministically related to downlink channel at time i, the challenge of acausality of knowledge has been scaled. Further (17) also ensures that the $\mathbf{H}_{\rm DL}$ is full-rank. If the internode interference channel at time i and j are same as well as (17) is satisfied, then BS can transmit 3 messages in two time slots, which results in the optimal downlink DoF of $3/2$.

Putting it all together: The success of the achievable scheme relies on how often we can find a time slot j complimentary to every time slot i, which has the same internode interference channel and also simultaneously satisfies (17). For channels drawn from continuous distribution exactly satisfying the two conditions is not possible. However, *approximately* satisfying the conditions is almost always possible (see [5] and [4] for details). Thus, DoF-pair $(3/2,3/2)$ is in fact achievable.

Remark 1: For any $L=K/2$, the strategy described in this section immediately achieves the only non-trivial corner point in the DoF-region. Furthermore, for arbitrary (L,K) the strategy achieves the symmetric DoF-pair $(\min(2L,K)/2,\min(2L,K)/2)$ and can be easily modified to achieve the other non-trivial corner points as well.

Remark 2: As is evident from the achievable strategy, the delay, i.e., time difference between i and j, incurred by the achievable strategy can be a major bottleneck in translating it into practice. Preliminary analysis of delay, for similar schemes like ergodic interference alignment in interference channel [6], shows that the delay grows exponentially in the number on individual channels between every pair of antennas in the network. For the (L,K) network, the number of individual channels is $2LK+K(K-1)$, which can make the delay prohibitively large for even small L and K.

V. THE $(2,2)$ FULL-DUPLEX NETWORK

In this section, we prove Theorem 2 and show that whenever the strength of internode interference channel is different relative to uplink/downlink channel, full-duplex has a larger GDoF-region than half-duplex $(2,2)$ network.

A. Achievability

1) $\beta<1$: In this regime, Theorem 2 tells us that there are two non-trivial corner points in the GDoF-region. Since $\beta<1$, interference is weaker than the signal of interest, where treating-interference-as-noise (TIN) is a commonly employed strategy for interference management. We show that for a $(2,2)$ network, in this interference regime TIN is optimal.

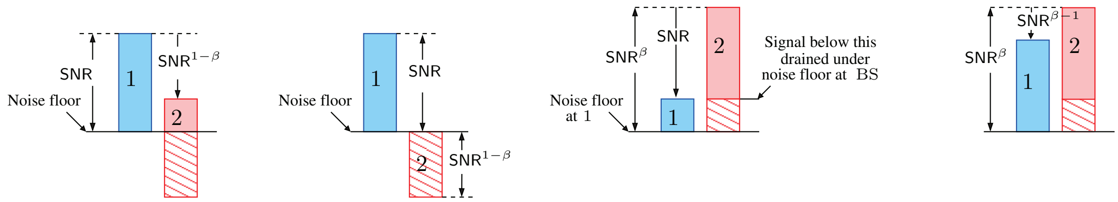
a) Corner point $(2,2(1-\beta))$: To achieve this corner point, both mobile users can transmit at a rate of $\log\mathsf{SNR}$ in uplink. To do so, they can transmit codewords $u_1[i]$ and $u_2[i]$ at time i, drawn from independent codebooks with rate $\log\mathsf{SNR}$ and power P. Since the base-station has 2 antennas, uplink decoding is trivial. Now, the uplink transmission from 1 will be received at 2 (and vice-versa) with an effective interference-to-noise ratio of SNR^{β} (see Fig. 3a). Due to power constraint at BS, the effective signal-to-noise ratio (SNR) of the intended signal at mobile receivers can be at most SNR, implying that the effective signal-to-interference-plus-noise ratio (SINR) will be $\mathsf{SNR}^{1-\beta}$. Therefore, BS can use transmit zero-forcing to communicate independent messages to each of the mobile users which are drawn from codebooks of rate $\log(\mathsf{SNR}^{1-\beta}/2)$ and power $P/2$. From the received signal, each mobile user can independently decode its intended message using TIN.

b) Corner point $(2(1-\beta),2)$: To achieve downlink GDoF of 2, BS can employ transmit zero-forcing and transmit independent downlink codewords to each user drawn from codebooks of rate $\log(\mathsf{SNR}/4)$ and power $P/2$. If the mobile users have to decode their intended messages using TIN, they should at least have an effective SINR of $\mathsf{SNR}/4$ which can be ensured if the mobile users transmit at a lower power, P/SNR^{β}. Then, the internode interference that the mobile users receive from each other will be at the same power level as thermal noise, as shown in Fig. 3b. If mobile users transmit at a lower power, they have to accordingly lower their uplink rate. Corresponding to a transmit power of P/SNR^{β}, each mobile user can encode its message at a rate $\log\mathsf{SNR}^{1-\beta}$ so that it is trivially decodable at BS.

2) $\beta\geq 1$: In this regime, the strength of the internode interference channel is stronger than the uplink/downlink channel which implies that decoding and cancelling interference from the received signal is a natural strategy for interference management. We show that in this regime decoding and cancelling interference is GDoF-optimal.

a) Corner point $(2,\min(2\beta-2,2))$: As usual, to achieve an uplink GDoF of 2, each mobile user can transmit independent codewords from codebooks of rate $\log(\mathsf{SNR}/2)$ and power P. The uplink transmissions will cause internode interference with an effective interference-to-noise ratio (INR) of SNR^{β}. Since $\beta>1$, the interference is completely decodable as long as the intended signal is received at an effective SNR of no more than $\mathsf{SNR}^{\beta-1}$. For any $\beta\leq 2$, the base-station has to ensure that it transmits at a lower power to allow the interference to remain decodable. Under interference decodability constraint, the base-station can transmit in downlink via transmit zero-forcing independent messages to each of the mobile users by picking codewords from independent codebooks of rate $\min(\log\mathsf{SNR},\log\mathsf{SNR}^{\beta-1})$ and corresponding power of $\min(P,P/\mathsf{SNR}^{2-\beta})$. Fig. 3c shows the relative power levels of the received signals at mobile user 1. From the received signal, the mobile users can decode the interference and subtract it out, and then they can decode the intended signal.

b) Corner point $(\min(2\beta-2,2),2)$: To achieve downlink GDoF of 2, BS can use transmit zero-forcing and transmit two independent codewords picked from independent codebooks of rate $\log(\mathsf{SNR}/2)$ and power $P/2$. Since the mobile users would like to decode the interference they observe, they



(a) Uplink signal from 2 is being transmitted at full power P

(b) Uplink signal from 2 is being transmitted at power P/SNR^{β}

(c) The transmit power of downlink signal from 1 is $P/\mathsf{SNR}^{\beta-1}$, while that of uplink signal from 2 is P

(d) The transmit power of downlink signal from 1 as well as the uplink signal from 2 is P

Fig. 3: The relative power levels of downlink signal intended for 1 and uplink signal from 2, as observed at the receiver of 1. Noise floor at the receiver is also depicted.

must transmit at a suitably low rate. If the mobile users transmit independent messages by picking codewords from codebooks of power P, then the effective interference-to-signal-plus-noise ratio is at least $\mathsf{SNR}^{\beta-1}/2$ (see Fig. 3d). For interference to be decodable, the mobile users have to pick codebooks of rate $\min(\log \mathsf{SNR}, \log(\mathsf{SNR}^{\beta-1}/2))$. The above choice of rate and power for uplink messages ensure their decodability both at the base-station as well as the interfering user. Upon decoding and cancelling the interference, the intended message can be easily decoded at the mobile users.

B. Outer-Bound

Proving (10c) of Theorem 2 proves the optimality of the achievability described in Section V-A. We prove it separately for regimes $\beta < 1$ and $\beta \geq 1$.

1) $\beta < 1$: To prove the upper bound on the sum-GDoF, we modify the existing network by making two changes. Firstly, the amplitude of the uplink and downlink channels is amplified by a factor $\mathsf{SNR}^{\frac{1-\beta}{2}}$. Secondly, the amplitude of internode channel is amplified by a factor of $\mathsf{SNR}^{1-\beta}$. We will show that the modification of the network achieved in the two steps together will have a capacity no less than the original network. To see this, let us revisit the input-output relationship for the modified network

$$\tilde{\mathbf{y}}_{\mathrm{BS}}[i] = \mathsf{SNR}^{\frac{1-\beta}{2}} \left(\mathbf{h}_{\mathrm{UL},1}[i]\tilde{u}_1[i] + \mathbf{h}_{\mathrm{UL},2}[i]\tilde{u}_2[i]\right) + \mathbf{z}_{\mathrm{BS}}[i] \tag{18a}$$

$$\tilde{y}_k[i] = \mathsf{SNR}^{\frac{1-\beta}{2}} \mathbf{h}^T_{\mathrm{DL},k}[i]\tilde{\mathbf{d}}[i] + \mathsf{SNR}^{1-\beta} h_{\mathrm{I},\bar{k}k}[i]\tilde{u}_{\bar{k}}[i] + z_k[i], \tag{18b}$$

where $\bar{k} = 2$ if $k = 1$ (and vice versa). In equation (18), we can set $\tilde{\mathbf{d}}[i] = \mathbf{d}[i]$, $\tilde{u}_k[i] = \frac{u_k[i]}{\sqrt{\mathsf{SNR}^{1-\beta}}}$ without sacrificing the original power constraint (assuming $\mathsf{SNR} \geq 1$) to obtain

$$\tilde{\mathbf{y}}_{\mathrm{BS}}[i] = \mathbf{h}_{\mathrm{UL},1}[i]u_1[i] + \mathbf{h}_{\mathrm{UL},2}[i]u_2[i] + \mathbf{z}_{\mathrm{BS}}[i] \tag{19a}$$

$$\tilde{y}_k[i] = \mathsf{SNR}^{\frac{1-\beta}{2}} \left(\mathbf{h}^T_{\mathrm{DL},k}[i]\mathbf{d}[i] + h_{\mathrm{I},\bar{k}k}[i]u_{\bar{k}}[i] + \frac{z_k[i]}{\mathsf{SNR}^{\frac{1-\beta}{2}}}\right). \tag{19b}$$

The input-output relationship described by equation (19a) is identical to the original input-output at BS described by equation (1a). Moreover, by simply adding additional independent noise to $\tilde{y}_k[i]$, output statistically equivalent to $y_k[i]$ can be obtained. Thus, the capacity of the channel described by equation (18) is no less than the capacity of channel described by equation (1). It is easy to verify that the new channel described by equation (18) has an effective uplink/downlink signal-to-noise ratio (SNR) of $\mathsf{SNR}^{2-\beta}$, and the effective strength of internode interference channel is also $\mathsf{SNR}^{2-\beta}$. From [4], we know that the sum-capacity (sum of uplink and downlink capacities) of a network where the strength of uplink/downlink channel and internode interference channel is same scales as $2\log \mathsf{SNR}$. Thus, the sum-capacity of the network described by (18) will scale as $2\log(\mathsf{SNR}^{2-\beta})$, which as $\mathsf{SNR} \to \infty$ results in a sum-GDoF of $2(2-\beta)$.

2) $\beta \geq 1$: We modify the existing network by amplifying the amplitude of the uplink/downlink channel by a factor $\mathsf{SNR}^{\frac{\beta-1}{2}}$. The resulting new input-output relationship is

$$\tilde{\mathbf{y}}_{\mathrm{BS}}[i] = \mathsf{SNR}^{\frac{\beta-1}{2}} \left(\mathbf{h}_{\mathrm{UL},1}[i]\tilde{u}_1[i] + \mathbf{h}_{\mathrm{UL},2}[i]\tilde{u}_2[i]\right) + \mathbf{z}_{\mathrm{BS}}[i] \tag{20a}$$

$$\tilde{y}_k[i] = \mathsf{SNR}^{\frac{\beta-1}{2}} \mathbf{h}^T_{\mathrm{DL},k}[i]\tilde{\mathbf{d}}[i] + h_{\mathrm{I},\bar{k}k}[i]\tilde{u}_{\bar{k}}[i] + z_k[i]. \tag{20b}$$

To show that the sum-capacity of the network described by (20) is no less than the sum-capacity of the original channel described by (1), we can set $\tilde{\mathbf{d}}[i] = \frac{\mathbf{d}[i]}{\sqrt{\mathsf{SNR}^{\beta-1}}}$, $\tilde{u}_k[i] = u_k[i]$ without sacrificing the original power constraint. Then, we get $\tilde{y}_k[i] = y_k[i]$. Since $\beta \geq 1$, from equation (20a) it is clear that we can obtain a statistically equivalent version of $\mathbf{y}_{\mathrm{BS}}[i]$ from $\tilde{\mathbf{y}}_{\mathrm{BS}}[i]$ by adding additional independent noise to $(\tilde{\mathbf{y}}_{\mathrm{BS}}[i])/\mathsf{SNR}^{\frac{\beta-1}{2}}$. Thus, the rates admissible by the original channel are also admissible by the new channel described by (20). It is easy to verify that in the new channel the effective uplink/downlink SNR is SNR^{β}, and the effective strength of internode interference channel is also SNR^{β}. Thus, the sum-capacity of channel described by (18) will scale as $2\log(\mathsf{SNR}^{\beta}) = 2\beta \log \mathsf{SNR}$, which implies that the sum-GDoF is upper bounded by 2β.

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