On Degrees-of-Freedom of Multi-User MIMO Full-Duplex Network

Jingwen Bai
Rice University, Houston, TX, USA
jingwen@rice.edu

Suhas Diggavi
UCLA, Los Angeles, CA, USA
suhas@ee.ucla.edu

Ashutosh Sabharwal
Rice University, Houston, TX, USA
ashu@rice.edu

***Abstract*—When a multi-antenna (MIMO) base-station operates in full-duplex mode, multiple uplink and downlink streams can be supported simultaneously in the same frequency band. However, the inter-mobile interference from uplink streams to the downlink streams can limit the system performance. In this paper, we first characterize the degrees-of-freedom of a multi-user MIMO (MU-MIMO) full-duplex network with half-duplex mobile clients, and derive the regimes where the inter-mobile interference can be mitigated to yield significant gains over the half-duplex counterpart. The achievability is based on interference alignment and requires full channel-state information at the transmitter (CSIT). Next, we study the case with partial CSIT where only the base-station acquires downlink channel values to avoid collecting network-wide CSIT at all transmitters in the system. We show that the key to achieving the sum degrees-of-freedom upper bound with only partial CSIT is the ability of the base-station to switch antenna modes that can be realized via reconfigurable antennas.**

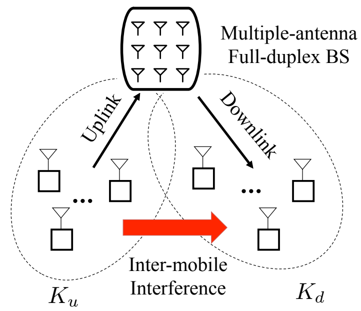

Fig. 1: M-antenna full-duplex BS supporting K_u uplink and K_d downlink single-antenna half-duplex mobile clients.

I. Introduction

In-band full-duplex wireless is an emerging technique, and promises to significantly improve the spectral efficiency, by allowing simultaneous transmission and reception in the same frequency spectrum. In fact, it is now feasible to design near-perfect full-duplex base-stations (e.g., see [1–3] and the references therein). In-band full-duplex has also become part of the ongoing standardization both in 3GPP [4] and 802.11-ax [5]. One potential way to leverage the full-duplex capability at the base-station (BS) is to support both uplink and downlink transmissions for legacy half-duplex mobile clients [6].

In this paper, we study a single-cell MU-MIMO full-duplex network, where an M-antenna full-duplex BS communicates with K_u uplink and K_d downlink streams in the same time-frequency slot. The space-constrained mobile devices are assumed to be single-antenna half-duplex. The simultaneous operation of uplink and downlink transmission, however, introduces inter-mobile interference from uplink users to downlink users as shown in Fig. 1. Therefore, we study the performance limits of the network labelled as (M, K_u, K_d) MU-MIMO full-duplex network, and identify the regimes where full-duplex operation is beneficial.

We first characterize the degrees-of-freedom (DoF) of (M, K_u, K_d) network. We show that when $\max(K_u, K_d) > M$, the full-duplex operation always yields gains over the half-duplex counterpart where uplink and downlink transmissions occur in orthogonal time/frequency slots. Our achievability is based on interference alignment [7] that requires full CSIT. We also study the case with partial CSIT, where only the BS needs to acquire downlink channel values. The key to achieving the sum DoF upper bound with only partial CSIT is the ability of BS to switch antenna modes which can be realized through reconfigurable antennas where more than one pattern can be radiated with different current distribution [8]. With antenna switching at BS, we artificially induce channel fluctuations for both up- and downlink channels that can be exploited to cancel out the inter-mobile interference without knowing the values of $K_u \times K_d$ interference channels in the network.

For related work, a full-duplex uplink/downlink channel is studied in [9] where all nodes are assumed to be full-duplex under ergodic phase fading. The use of antenna mode switching has been investigated in [8] at the mobile clients to achieve blind interference alignment for MIMO broadcast channels. In this paper, we leverage the antenna-switching ability at the full-duplex BS for up- and downlink channels with half-duplex mobile clients. At the completion of this work, we were informed of [10] (posted concurrently to our submission) which independently and simultaneously considered a similar problem formulation. From our understanding, there are a few differences including that we studied the DoF region and considered the use of antenna modes at the BS to reduce the need for all transmitters to acquire network-wide CSIT.

II. System Model

In a single-cell (M, K_u, K_d) MU-MIMO full-duplex network, the full-duplex capability is available only at the BS

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so that the BS can use M antennas to serve both uplink and downlink users in the same time-frequency slot. The mobile users are half-duplex with single antenna. We start by presenting the system input-output relationship.

The received uplink signal at the BS at time index t is $\mathbf{y}_u(t) \in \mathbb{C}^{M\times 1}$ and is given as

$$\mathbf{y}_u(t) = \sum_{i=1}^{K_u} \mathbf{h}_{u,i}(t) x_{u,i}(t) + \mathbf{z}_u(t),$$

where $\mathbf{h}_{u,i}(t) \in \mathbb{C}^{M\times 1}$ is the uplink channel from user i to BS and $x_{u,i}(t)$ is the transmit signal of uplink user i; $\mathbf{z}_u(t) \in \mathbb{C}^{M\times 1}$ is the receiver thermal noise which contains independent and identically distributed (i.i.d.) circularly-symmetric complex Gaussian distributed $\mathcal{CN}(0,1)$ entries.

The received signal at the j-th downlink user $y_{d,j}(t)$ is a combination of the downlink signal and the interfering uplink signals, and is given by

$$y_{d,j}(t) = \mathbf{h}_{d,j}(t)\mathbf{x}_d(t) + \sum_{i=1}^{K_u} g_{j,i}(t) x_{u,i}(t) + z_{d,j}(t),$$

where $\mathbf{h}_{d,j}(t) \in \mathbb{C}^{1\times M}$ is the downlink channel from BS to downlink user j and $\mathbf{x}_d(t) \in \mathbb{C}^{M\times 1}$ is the transmit signal vector of BS; $g_{j,i}(t)$ is the inter-mobile interference channel from uplink user i to downlink user j. The receiver noise $z_{d,j}(t)$ follows $\mathcal{CN}(0,1)$.

All the channel coefficients are drawn i.i.d. from a continuous distribution. Each transmitter satisfies the average power constraint such that $\mathbb{E}(|x_{u,i}(t)|^2) \le P$, $\forall i \in \{1,\cdots,K_u\}$ and $\mathbb{E}(\mathbf{x}_d(t)^\dagger \mathbf{x}_d(t)) \le P$.

We define the encoding function at the BS so that the codeword of downlink message at time t is $\mathbf{x}_d(t) = f_d^t(W_{d,1},\cdots,W_{d,K_d},\mathbf{y}_u^{t-1})$, where $\mathbf{y}_u^{t-1} \triangleq (\mathbf{y}_u(1),\cdots,\mathbf{y}_u(t-1))$. The encoding function at the i-th uplink user is $x_{u,i}(t) = f_{u,i}^t(W_{u,i})$, $\forall i \in \{1,\cdots,K_u\}$. The uplink and downlink rate tuple $(R_{u,1},\cdots,R_{u,K_u})$ and $(R_{d,1},\cdots,R_{d,K_d})$ are said to be achievable if coding over t_0 channel uses, we obtain $\text{Prob}(\hat{W}_{u,i} \neq W_{u,i}) \to 0$ and $\text{Prob}(\hat{W}_{d,j} \neq W_{d,j}) \to 0$ as $t_0 \to \infty$.

Next, we define the achievable degrees-of-freedom of the uplink and downlink as

$$\mathsf{DoF}_\theta = \lim_{P\to\infty} \frac{\sum_{i=1}^{K_\theta} R_{\theta,i}(P)}{\log P}, \ \theta \in \{u,d\}. \tag{1}$$

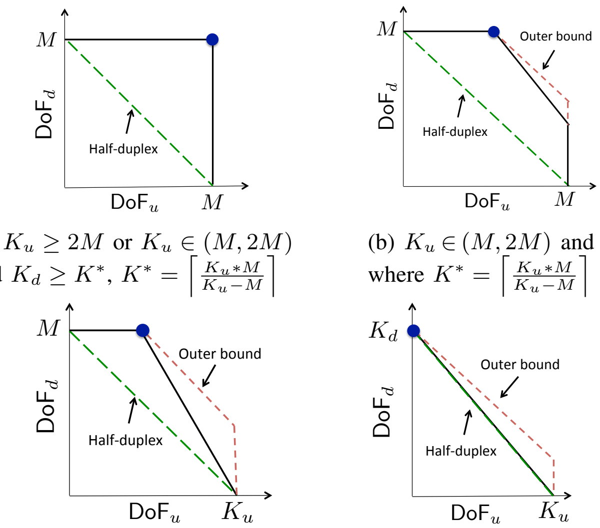
The achievable sum DoF can also be computed as

$$\mathsf{DoF}_{sum} = \mathsf{DoF}_u + \mathsf{DoF}_d \tag{2}$$

The optimal sum DoF is the maximum of DoF_{sum}, and is denoted as DoF^*_{sum}.

III. Main Results

The results with full CSIT are first stated in Theorems 1 and 2. Next, we present the results with partial CSIT when BS has reconfigurable antennas in Theorem 3. We assume the BS can switch among S preset modes [8], which will not affect our DoF outer bound. Each single antenna at the BS can select an antenna mode from S possible antenna configuration patterns. Different modes selection at the BS will lead to independent up- and downlink channel realizations, respectively, by inducing artificial channel variations.



(a) $K_u \ge 2M$ or $K_u \in (M,2M)$ and $K_d \ge K^*$, $K^* = \left\lceil \frac{K_u * M}{K_u - M} \right\rceil$

(b) $K_u \in (M,2M)$ and $K_d < K^*$, where $K^* = \left\lceil \frac{K_u * M}{K_u - M} \right\rceil$

(c) $K_d > M$ and $K_u \le M$

(d) $K_d \le M$

Fig. 2: DoF region of (M,K_u,K_d) full-duplex network for $K_d \ge K_u$. The blue point denotes the DoF^*_{sum}.

Theorem 1 (Interference Alignment with Full CSIT). The achievable uplink and downlink DoF region (DoF_u, DoF_d) has the following corner points:

$$\begin{aligned}
&i) && (\min(M,K_u),0),\\
&ii) && (0,\min(M,K_d)),\\
&iii) && \left(\min(M,K_u), \min\left(\frac{K_d}{K_u}(K_u-M)^+, M\right)\right),\\
&iv) && \left(\min\left(\frac{K_u}{K_d}(K_d-M)^+, M\right), \min(M,K_d)\right),
\end{aligned}$$

where $(x)^+ = \max(0,x)$.

Remark 1. Theorem 1 demonstrates that when $\max(K_u,K_d) > M$, full-duplex always outperforms the half-duplex system in the achievable DoF region as depicted in Fig. 2. Specifically, when $\min(K_u,K_d) \ge 2M$, or when $\min(K_u,K_d) \in (M,2M)$ and $\max(K_u,K_d) \ge K^*$, where $K^* = \left\lceil \frac{\min(K_u,K_d)*M}{\min(K_u,K_d)-M} \right\rceil$, we can achieve the maximum square region that doubles the region of the half-duplex counterpart. Fig. 2 shows the DoF region under different antenna and user number setups for $K_d \ge K_u$.

Remark 2. The inner and outer bounds on the DoF region (DoF_u, DoF_d) are tight when $K_u = K_d$, where the outer bound is given in Lemma 1 (see Section IV-B).

Theorem 2. The optimal sum DoF of (M,K_u,K_d) MU-MIMO full-duplex network is completely characterized as:

$$\mathsf{DoF}^*_{sum} = \min(\Delta, 2M), \tag{3}$$

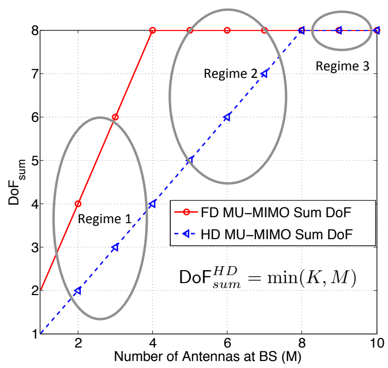


Fig. 3: The optimal sum DoF of full-duplex system as a function of the number of BS antennas when $K_u = K_d = 8$.

where $\Delta = \frac{\min(M,K_u)(K_u-K_d)^{+}+\min(M,K_d)(K_d-K_u)^{+}+K_uK_d}{\max(K_u,K_d)}$.

Remark 3. From Theorem 2, $\mathsf{DoF}^{*}_{sum} = \min(K, 2M)$ when $K_u = K_d = K$. Fig. 3 shows DoF^{*}_{sum} as a function of the number of BS antennas, and compares the optimal sum DoF of full-duplex with that of half-duplex system when $K = 8$. We can see that there are three regimes in comparison: 1) when $K \geq 2M$, full-duplex system achieves twice the sum DoF as half-duplex counterpart; 2) when $M < K < 2M$, the gain of full-duplex over half-duplex is $\frac{K}{M}$-fold, and reduces as the number of BS antennas increases; 3) when $K \leq M$, there is no benefit due to full-duplex operation. Hence full-duplex gain is pronounced with a large number of users in the network. We also observe from Fig. 3 that the half-duplex system with a larger number of BS antennas can achieve the same DoF^{*}_{sum} of the full-duplex system with less BS antennas.

Theorem 3 (Antenna Switching with Partial CSIT). Assuming the BS has reconfigurable antenna array with S preset modes, the sum DoF upper bound is still achievable with partial CSIT, i.e., only the BS acquires downlink channel values. The required number of antenna modes s^* ($s^* \leq S$) is [1]:

i. when $K_u = K_d$ or when $\min(K_u, K_d) \geq 2M$, $s^* = 2$;

ii. otherwise, the required s^* should satisfy the following feasibility condition, where $b \in \mathbb{N}, s^* \in \mathbb{N}^{+}, b \leq s^* \leq S$:

$$\frac{b}{s^*} = \begin{cases} \frac{M}{K_d}, & \text{if } K_d > K_u \\ 1 - \frac{M}{K_u}, & \text{if } K_u > K_d \end{cases} \tag{4}$$

Remark 4. Theorem 3 shows that when we have a larger number of users than number of BS antennas, or equal number of up- and downlink users, BS can leverage two antenna modes such as horizontal and vertical polarizations to achieve DoF^{*}_{sum} with only partial CSIT.

IV. Interference Alignment with Full CSIT

A. Achievability

Our achievable scheme with full CSIT is based on interference alignment [7] under time varying channels. The main idea is to align all the inter-mobile interference at each downlink receiver within a small dimension, and use the remaining interference-free dimension to decode downlink messages which are transmitted via zero-forcing beamforming at the BS. The generic uplink channel values ensures the BS can decode all the uplink messages almost surely via receive zero-forcing beamforming.

We let $N = (K_u - 1)K_d$. We show that $(\mathsf{DoF}_u, \mathsf{DoF}_d) = \left(\alpha\frac{(n+1)^N+(K_u-1)n^N}{(n+1)^N}, \beta\frac{K_d n^N}{(n+1)^N}\right)$ lies in the achievable DoF region $\forall n \in \mathbb{N}$. This implies that

$$\mathsf{DoF}_u \to \alpha K_u,\ \mathsf{DoF}_d \ \to \beta K_d,\ \text{as } n \to \infty, \tag{5}$$

where $\alpha + \beta \in (0, 1], \alpha K_u \leq M, \beta K_d \leq M, \alpha, \beta \in (0, 1]$.

We will construct an interference alignment scheme to show that $\left((n+1)^N + (K_u-1)n^N, \frac{\beta}{\alpha}K_d n^N\right)$ is achievable of an $D_n = \frac{1}{\alpha}(n+1)^N$ symbol extension of the channel, which will lead to the above result. For the D_n-extended channel, the received uplink and downlink signals are given as follows:

$$\overline{\mathbf{Y}}_u(t) = \sum_{i=1}^{K_u} \overline{\mathbf{H}}_{u,i}(t)\overline{\mathbf{X}}_{u,i}(t) + \overline{\mathbf{Z}}_u(t),$$

$$\overline{\mathbf{Y}}_{d,j}(t) = \overline{\mathbf{H}}_{d,j}(t)\overline{\mathbf{X}}_d(t) + \sum_{i=1}^{K_u} \overline{\mathbf{G}}_{j,i}(t)\overline{\mathbf{X}}_{u,i}(t) + \overline{\mathbf{Z}}_{d,j}(t),$$

where [2]

$$\begin{aligned} \overline{\mathbf{H}}_{u,i}(t) &= diag(\mathbf{h}_{u,i}[D_n(t-1)+1], \cdots, \mathbf{h}_{u,i}[D_n t]), \\ \overline{\mathbf{H}}_{d,j}(t) &= diag(\mathbf{h}_{d,i}[D_n(t-1)+1], \cdots, \mathbf{h}_{d,i}[D_n t]), \\ \overline{\mathbf{G}}_{j,i}(t) &= diag(g_{j,i}[D_n(t-1)+1], \cdots, g_{j,i}[D_n t]), \\ \overline{\mathbf{Z}}_u(t) &= [\mathbf{z}_{ul}[D_n(t-1)+1], \cdots, \mathbf{z}_u[D_n t]]^{\dagger}, \\ \overline{\mathbf{Z}}_{d,j}(t) &= [z_{dl,j}[D_n(t-1)+1], \cdots, z_{d,j}[D_n t]]^{\dagger}. \end{aligned} \tag{6}$$

In the D_n-extended channel, uplink message $W_{u,1}$ will first be encoded into $(n+1)^N$ independent streams $x^{[k]}_{u,1}$, $k = 1, \cdots, , (n+1)^N$. Each codeword is drawn from i.i.d. Gaussian codebook according to $\mathcal{CN}(0, P)$. Each codeword will be transmitted along vector $\mathbf{v}^{[k]}_{u,1}$ as follows:

$$\overline{\mathbf{X}}_{u,1} = \gamma_1 \sum_{k=1}^{(n+1)^N} \mathbf{v}^{[k]}_{u,1} x^{[k]}_{u,1} = \gamma_1 \overline{\mathbf{V}}_{u,1}\mathbf{X}_{u,1},$$

where $\overline{\mathbf{V}}_{u,1} \in \mathbb{C}^{D_n \times (n+1)^N}, \mathbf{X}_{u,1} \in \mathbb{C}^{(n+1)^N \times 1}$; $\gamma_1 > 0$ is chosen to satisfy the power constraint.

Likewise, uplink user $i \in \{2, \cdots, K_u\}$ will encode message $W_{u,i}$ into n^N independent streams as follows:

$$\overline{\mathbf{X}}_{u,i} = \gamma_1 \sum_{k=1}^{n^N} \mathbf{v}^{[k]}_{u,i} x^{[k]}_{u,i} = \gamma_1 \overline{\mathbf{V}}_{u,i}\mathbf{X}_{u,i},$$

where $\overline{\mathbf{V}}_{u,i} \in \mathbb{C}^{D_n \times n^N}, \mathbf{X}_{u,i} \in \mathbb{C}^{n^N \times 1}$.

For the downlink transmission, the BS encodes the j-th downlink user's message $W_{d,j}$ into $\frac{\beta}{\alpha}n^N$ independent streams

[1] We assume that $\max(K_u, K_d) > M$, otherwise, half-duplex system already achieves the DoF^{*}_{sum} which requires partial CSIT.

[2] In the rest of the paper, we will omit t for the D_n-extended channel.

using i.i.d. Gaussian codebook according to $\mathcal{CN}(0, P)$. The BS will transmit the downlink streams along vectors $\mathbf{v}_{d,j}^{[k]}$:

$$\overline{\mathbf{X}}_d = \gamma_2 \sum_{j=1}^{K_d} \sum_{k=1}^{\frac{\beta}{\alpha}n^N} \mathbf{v}_{d,j}^{[k]} x_{d,j}^{[k]} = \gamma_2 \sum_{j=1}^{K_d} \overline{\mathbf{V}}_{d,j} \mathbf{X}_{d,j},$$

where $\overline{\mathbf{V}}_{d,j} \in \mathbb{C}^{MD_n \times \frac{\beta}{\alpha}n^N}, \mathbf{X}_{d,j} \in \mathbb{C}^{\frac{\beta}{\alpha}n^N \times 1}$; $\gamma_2 > 0$ is chosen to satisfy the power constraint.

The received signal at the j-th downlink user is

$$\overline{\mathbf{Y}}_{d,j} = \overline{\mathbf{H}}_{d,j} \overline{\mathbf{X}}_d + \sum_{i=1}^{K_u} \gamma_1 \overline{\mathbf{G}}_{j,i} \overline{\mathbf{V}}_{u,i} \mathbf{X}_{u,i} + \overline{\mathbf{Z}}_{d,j},$$

The signal space for the j-th downlink user is $span\left(\{\overline{\mathbf{H}}_{d,j}\overline{\mathbf{V}}_{d,j}\}\right)$ which occupies $\frac{\beta}{\alpha}n^N$-dimensional subspace in D_n-dimensional space almost surely. For the downlink transmission, the BS will design the zero-forcing beamforming vector satisfying that

$$\overline{\mathbf{H}}_{dl,j'} \overline{\mathbf{V}}_{dl,j'} \perp span\left(\{\overline{\mathbf{H}}_{d,j}\overline{\mathbf{V}}_{d,j}\}\right), \tag{7}$$

where $j \neq j' \in \{1, \cdots, K_d\}$. The total dimension at the BS is MD_n, and the nulling is successful if

$$\frac{M}{\alpha}(n+1)^N - (K_d - 1)\frac{\beta}{\alpha}n^N \geq \frac{\beta}{\alpha}n^N, \tag{8}$$

and we have $\beta K_d \leq M$ as $n \to \infty$.

Each downlink user now needs to extract $\frac{\beta}{\alpha}n^N$ interference-free dimensions from a D_n-dimensional received signal vector to decode its signals by zero-forcing all inter-mobile interference from uplink users. With the interference alignment vectors constructed in a similar way as in [7], we can ensure the inter-mobile interference occupies at most $(n+1)^N$ signal dimension almost surely. To ensure the overall dimension is large enough to accommodate both signal and inter-mobile interference dimension, the following condition has to be satisfied:

$$\frac{\beta}{\alpha}n^N + (n+1)^N \leq \frac{1}{\alpha}(n+1)^N, \tag{9}$$

and we have $\alpha + \beta \leq 1$ as $n \to \infty$.

Next, we need to ensure that downlink signals are linearly separable from the inter-mobile interference dimension so that the desired signals can be decoded. It suffices to show that the following matrix

$$\left[\ \overline{\mathbf{H}}_{d,j}\overline{\mathbf{V}}_{d,j} \quad \overline{\mathbf{G}}_{ji}\overline{\mathbf{V}}_{u,i}\ \right] \tag{10}$$

is full rank almost surely. Firstly, $\overline{\mathbf{V}}_{d,j}$ is independent of $\overline{\mathbf{H}}_{d,j}$ since $\overline{\mathbf{V}}_{d,j}$ is a function of $\overline{\mathbf{H}}_{dl,j'}, j' \neq j$ from (7), and $\overline{\mathbf{H}}_{d,j}$ is generic. Hence all column vectors in $\overline{\mathbf{H}}_{d,j}\overline{\mathbf{V}}_{d,j}$ are linearly independent almost surely since $\frac{\beta}{\alpha}n^N \leq D_n$. Furthermore, the choice of $\overline{\mathbf{V}}_{d,j}$ is also independent of $\overline{\mathbf{G}}_{j,i}\overline{\mathbf{V}}_{u,i}, \forall i, j$. Thus the above matrix contains linearly independent column vectors almost surely. Therefore, we can deliver total $K_d\frac{\beta}{\alpha}n^N$ independent downlink streams.

For the uplink reception at the BS, the choice of $\overline{\mathbf{V}}_{u,i}$ is independent of $\overline{\mathbf{H}}_{u,i}$ as $\overline{\mathbf{V}}_{u,i}$ is a function of $\overline{\mathbf{G}}_{j,i}, \forall i, j$ based on our construction of the interference alignment vectors similar to [7]. Therefore all column vectors of the set of matrices $\{\overline{\mathbf{H}}_{u,i}\overline{\mathbf{V}}_{u,i}\}_{i \in \{1, \cdots, K_u\}}$ are linearly independent almost surely if

$$(n+1)^N + (K_u - 1)n^N \leq MD_n, \tag{11}$$

and we have $\alpha K_u \leq M$ as $n \to \infty$. We can now deliver total $(n+1)^N + (K_u - 1)n^N$ independent uplink streams.

Thus we show that $\left(\alpha\frac{(n+1)^N + (K_u-1)n^N}{(n+1)^N}, \beta\frac{K_d n^N}{(n+1)^N}\right)$ lies in the achievable DoF region satisfying (8), (9) and (11).

We will verify that all the corner points in Theorem 1 are achievable. The achievability of corner point (i) and (ii) is trivial. In order to achieve (iii), we choose α, β as follows to meet the conditions in (8), (9) and (11): $\alpha = \min\left(\frac{M}{K_u}, 1\right), \beta = \min\left(1 - \alpha, \frac{M}{K_d}\right) = \min\left(\frac{(K_u - M)^+}{K_u}, \frac{M}{K_d}\right)$, where $(x)^+ = \max(x, 0)$. Now by substituting our choice of α, β into (5), we can achieve (iii).

For corner point (iv), we will set $\beta = \min\left(\frac{M}{K_d}, 1\right)$, $\alpha = \min\left(1 - \beta, \frac{M}{K_u}\right) = \min\left(\frac{(K_d - M)^+}{K_d}, \frac{M}{K_u}\right)$, and we can show that (iv) is achievable with such choice of α and β. Hence we have proved Theorem 1.

The sum DoF is $\alpha K_u + \beta K_d$ as $n \to \infty$, and the lower bound on DoF_{sum}^* can be derived by solving the following optimization problem.

$$\begin{aligned} &\max_{\alpha,\beta} \alpha K_u + \beta K_d \\ &\text{s.t.}\ \ 0 < \alpha + \beta \leq 1 \\ &\quad \alpha K_u \leq M,\ \beta K_d \leq M \end{aligned} \tag{12}$$

The above convex problem can be solved easily which leads to $\mathsf{DoF}_{sum}^* \geq \min(2M, \Delta)$, where Δ is given in (3).

B. Converse

Lemma 1. The outer bound on (M, K_u, K_d) MU-MIMO full-duplex network is given as

$$\mathsf{DoF}_u \leq \min(M, K_u) \tag{13}$$
$$\mathsf{DoF}_d \leq \min(M, K_d) \tag{14}$$
$$\mathsf{DoF}_{sum} \leq \min(2M, \Delta) \tag{15}$$

Proof. The individual point-to-point upper bound is trivial. The sum DoF upper bound follows similar to [9, 11] and the details are omitted due to space constraint. □

With the achievability in (12), we prove that $\mathsf{DoF}_{sum}^* = \min(2M, \Delta)$ as stated in Theorem 2.

V. Antenna Switching with Partial CSIT

In this section, we will show how we can still achieve the sum DoF upper bound with partial CSIT where only the BS needs to acquire downlink channel values.

The key to achieving DoF_{sum}^* with only partial CSIT is the ability of BS to switch antenna modes. We assume the antenna array at the BS has S preset modes. The BS will switch the antenna modes to artificially induce short term up- and downlink channel fluctuation patterns that can be exploited to manage inter-mobile interference. We assume the BS will learn the downlink channels for all antenna modes prior to the start of downlink transmissions.

The channel coherence times are assumed to be long enough such that the all channels remain constant across S-symbol extended channels. Now we will describe the achievability. During the s-symbol ($s \leq S$) communication period, BS will switch antenna modes at each symbol period, and the up- and downlink channels will vary independently from different modes, while the inter-mobile interference channel values remain unchanged. Each uplink signal is transmitted as $\tilde{\mathbf{x}}_{u,i} = \gamma_1 \mathbf{F}_i \mathbf{u}_i$, where $\mathbf{u}_i = [u_{i,1}, \cdots, u_{i,s-b}]^\dagger \in \mathbb{R}^{(s-b)\times 1}$ is the i-th uplink user's message, $b \in \mathbb{N}, s \in \mathbb{N}^+, b \leq s \leq S$; $\mathbf{F}_i$ is the associated precoding matrix, which is independent of the channel gains and is given as

$$\mathbf{F}_i = \begin{pmatrix} \mathbf{I}_{s-b} \\ \tilde{\mathbf{V}}_{b,s-b} \end{pmatrix}.$$

$\tilde{\mathbf{V}}_{b,s-b}$ is a $b \times (s-b)$ Vandermonde matrix:

$$\tilde{\mathbf{V}}_{b,s-b} = \begin{pmatrix} \omega_1 & \omega_1^2 & \cdots & \omega_1^{s-b} \\ \omega_2 & \omega_2^2 & \cdots & \omega_2^{s-b} \\ \vdots & \vdots & \ddots & \vdots \\ \omega_b & \omega_b^2 & \cdots & \omega_b^{s-b} \end{pmatrix},$$

where $\omega_k = e^{-j\frac{2\pi}{s+1}k}, k \in \{1, \cdots, b\}$. Hence columns in $\mathbf{F}_i$ are linearly independent of each other.

The i-th uplink user's transmit signal is thus expressed as

$$\tilde{\mathbf{x}}_{u,i} = \begin{bmatrix} x_{u,i}(1) \\ \vdots \\ x_{u,i}(s) \end{bmatrix} = \gamma_1 \begin{bmatrix} \mathbf{u}_i \\ f_1(\mathbf{u}_i) \\ \vdots \\ f_b(\mathbf{u}_i) \end{bmatrix}, \quad (16)$$

where $f_k(\mathbf{u}_i) = \sum_{p=1}^{s-b} \omega_k^p u_{i,p}, k \in \{1, \cdots, b\}$. We can see that each transmit signal of uplink user i in the last b-symbol time slots is a linear combination of the transmit signals during the first $(s-b)$-symbol time slots.

The s-symbol extended received uplink signal at BS is $\tilde{\mathbf{y}}_u = \sum_{i=1}^{K_u} \tilde{\mathbf{H}}_{u,i} \tilde{\mathbf{x}}_{u,i} + \tilde{\mathbf{z}}_u$, where $\tilde{\mathbf{H}}_{u,i} = diag\left(\mathbf{h}_{u,i}(1), \cdots, \mathbf{h}_{u,i}(s)\right)$, $\tilde{\mathbf{z}}_u = [\mathbf{z}_u(1), \cdots, \mathbf{z}_u(s)]^\dagger$.

At the decoding of uplink message, $\left\{\tilde{\mathbf{H}}_{u,i}\mathbf{F}_i\right\}_{\forall i \in \{1, \cdots, K_u\}}$ contains a set of linearly independent column vectors almost surely if $K_u(s-b) \leq Ms$. Thus over s-symbol period, we can deliver $\mathsf{DoF}_u = \min\left(\frac{K_u(s-b)}{s}, M\right)$ almost surely.

For the j-th downlink user, the received signal during the s-symbol time slots is

$$y_{d,j}(\kappa) = \mathbf{h}_{d,j}(\kappa)\mathbf{x}_d(\kappa) + \sum_i g_{j,i} x_{u,i}(\kappa) + z_{d,j}(\kappa), \quad (17)$$

where $\kappa = \{1, \cdots, s\}$.

Since the interference channel $g_{j,i}, \forall i, j$ remains constant during s-symbol period, the downlink user j will perform the following subtraction from the received signals:

$$y_{d,j}(q) - \sum_{p=1}^{s-b} \omega_k^p y_{d,j}(p) = \mathbf{h}_{d,j}(q)\mathbf{x}_d(q) - \sum_{p=1}^{s-b} \omega_k^p \mathbf{h}_{d,j}(p)\mathbf{x}_{d,j}(p), \quad (18)$$

where $k = q - (s-b)$ and $q = \{s-b+1, \cdots, s\}$. Here we ignore the noise term since it does not affect the DoF analysis.

We let $r_j(q) = y_{d,j}(q) - \sum_{p=1}^{s-b} \omega_k^p y_{d,j}(p)$, and we denote $\mathbf{r}'_j = [r_j(s-b+1), \cdots, r_j(s)]^\dagger$. Now we can rewrite (18) as

$$\mathbf{r}'_j = \mathbf{H}_{\text{eff},j} \tilde{\mathbf{x}}_d, \quad (19)$$

where $\tilde{\mathbf{x}}_d = [\mathbf{x}_d(1), \cdots, \mathbf{x}_d(s)]$, and $\mathbf{H}_{\text{eff},j} \in \mathbb{R}^{b \times Ms}$ is the effective j-th downlink channel during s-symbol period which can be obtained from (18).

The j-th downlink user's message $\mathbf{d}_j = [d_{j,1}, \cdots, d_{j,b}]^\dagger \in \mathbb{R}^{b\times 1}$ will be transmitted with a s-symbol extended beamforming matrix $\mathbf{E}_j \in \mathbb{R}^{Ms \times b}$ and the BS will transmit $\tilde{\mathbf{x}}_d = \gamma_2 \sum_{j=1}^{K_d} \mathbf{E}_j \mathbf{d}_j$.

The BS will design the zero-forcing beamforming matrix to null out inter-beam interference for the downlink transmission. Following the same analysis as in Section IV-A, each downlink user can decode b messages almost surely if $bK_d \leq Ms$. Thus we can deliver $\mathsf{DoF}_d = \min\left(\frac{bK_d}{s}, M\right)$ almost surely.

Hence the achievable sum DoF with partial CSIT is

$$\begin{aligned} \mathsf{DoF}_{sum}^{\text{Partial}} = \max_{b,s} & \left\{ \min\left(\frac{K_u(s-b)}{s}, M\right) + \min\left(\frac{bK_d}{s}, M\right)\right\} \\ \text{s.t. } & b \in \mathbb{N}, s \in \mathbb{N}^+, \\ & b \leq s \leq S. \end{aligned} \quad (20)$$

Lemma 2. The DoF outer bound in Lemma 1 still holds given the antenna mode switching ability at the BS.

Proof. The converse still holds because the assumptions made for the DoF outer bound will not be affected by antenna mode switching at the BS. The details are omitted due to space limit. □

By comparing $\mathsf{DoF}_{sum}^{\text{Partial}}$ in (20) with the sum DoF upper bound in (15), we can prove Theorem 3.

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