Interference Channels with Bursty Traffic and Delayed Feedback

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***Abstract*—In this paper we study interference management in wireless networks with bursty user traffic. In each time slot, whether a user is on or off for transmission is governed by its own Bernoulli random state. At each transmitter, the states of activities of other users are only available via feedback. We investigate a canonical two-user bursty Gaussian interference channel (IC) with three different feedback models: (1) no feedback, (2) delayed state feedback, and (3) channel output feedback. In all three cases, we characterize the capacity region of the bursty Gaussian IC to within a bounded gap. It turns out that the near-optimal transmit strategies in the non-bursty IC suffice to establish the approximate characterization of capacity in all three cases. In other words, traffic burstiness does not change the high-SNR optimality of the schemes as long as receivers keep track of user activities. Moreover, the capacity region with delayed state feedback is within a bounded gap to that without feedback, and therefore delayed state feedback does not provide significant improvement at high SNR.**

I. INTRODUCTION

Interference limits the performance of modern wireless networks, and hence understanding the fundamental limits of interference networks serves as a first crucial step towards optimal interference management. Recent advances in network information theory make great strides in this direction: despite that the capacity region characterization of the two-user interference channel (IC) remains open, that of the two-user Gaussian IC is characterized to within 1 bit/s/Hz [1]. Moreover, when channel output feedback is available, the capacity region is characterized to within 2 bits/s/Hz [2]. These approximate capacity results shed light on how to optimally manage interference at high SNR with and without feedback.

In most literatures regarding interference networks, it is usually assumed that interference is always present, either associated with static or fading channel coefficients. In many realistic scenarios, however, wireless traffic is *bursty* owing to the decentralized networking protocol across different users: users may not transmit their own information all the time, and hence in the physical layer, it may be too pessimistic to assume that interference is always present. A natural way to exploit such burstiness is the degraded-message-set approach proposed in [3], where each receiver opportunistically decodes an additional message when there is no interference. When there is interference, it only decodes the original message, not the additional one. Such a degraded-message-set approach, however, does not exploit the opportunity to code over multiple blocks of transmission. Moreover, it does not make use of feedback, which is present in most wireless systems. Potentially these two elements could enhance system capacity.

In this work, we study interference management in wireless networks with bursty user traffic. In particular, the opportunities of coding over multiple transmission blocks as well as feedback from the receivers to the respective transmitters are exploited. We focus on a two-user bursty Gaussian IC, where for user i, $i = 1, 2$, a stationary and ergodic Bernoulli process $\{S_i[t]\}$ controls whether the user is active or not. The realization of the *burstiness states* $(S_1[t], S_2[t])$ is known to both receivers at the end of time slot t and can be viewed as part of the channel output. Three different feedback models are considered: (1) no feedback, (2) delayed state feedback, where the other user's state is known to the transmitter with unit delay, and (3) channel output feedback, where the whole channel output is known to the transmitter with unit delay.

Our main contribution is the approximate characterization of the capacity regions of the two-user bursty Gaussian IC in the above three settings. Noting that the presence of one user's signal at both receivers is governed by a single state (its own state), we show that the schemes in the non-bursty case, that is, the Han-Kobayashi scheme in the non-feedback case [4] [1] and the Suh-Tse scheme in the feedback case [2], suffice to achieve the capacity regions to within a bounded gap. In the cases of non-feedback and delayed state feedback, each user employs the coding scheme described in [1] with the same power split. In the case of channel output feedback, each user employs the block Markov coding scheme along with backward decoding as in [2]. Outer bounds are proved by a genie providing non-causal state information to the transmitters, together with the techniques used in the non-bursty case [1] [2]. Our main result suggests that interference management in the two-user IC is resilient to the burstiness of user traffic. The reason is that there is no *mismatch* in the presence of signals at the two receivers – whenever the signal is present at one receiver, it is also present at the other. Therefore burstiness does not change the problem significantly compared with the non-bursty case, and we can apply the non-bursty coding schemes directly.

The rest of this paper is organized as follows. Section II and III formulates the problem and states the main result respectively. Section IV presents proofs for the case without feedback and with delayed state feedback. Section V proves the result in the case with channel output feedback.

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$$R_1 \le \gamma_1 \mathsf{C}(\mathsf{SNR}_1) \tag{1}$$
$$R_2 \le \gamma_2 \mathsf{C}(\mathsf{SNR}_2) \tag{2}$$
$$R_1+R_2 \le \gamma_1 \mathsf{C}\left(\tfrac{\mathsf{SNR}_1}{1+\mathsf{INR}_2}\right) + q_{11}\mathsf{C}(\mathsf{SNR}_2+\mathsf{INR}_2) + q_{01}\mathsf{C}(\mathsf{SNR}_2) + q_{10}\mathsf{C}(\mathsf{INR}_2) \tag{3}$$
$$R_1+R_2 \le \gamma_2 \mathsf{C}\left(\tfrac{\mathsf{SNR}_2}{1+\mathsf{INR}_1}\right) + q_{11}\mathsf{C}(\mathsf{SNR}_1+\mathsf{INR}_1) + q_{10}\mathsf{C}(\mathsf{SNR}_1) + q_{01}\mathsf{C}(\mathsf{INR}_1) \tag{4}$$
$$R_1+R_2 \le q_{11}\left\{\mathsf{C}\left(\mathsf{INR}_1+\tfrac{\mathsf{SNR}_1}{1+\mathsf{INR}_2}\right) + \mathsf{C}\left(\mathsf{INR}_2+\tfrac{\mathsf{SNR}_2}{1+\mathsf{INR}_1}\right)\right\} + q_{10}\mathsf{C}(\mathsf{SNR}_1+\mathsf{INR}_2) + q_{01}\mathsf{C}(\mathsf{SNR}_2+\mathsf{INR}_1) \tag{5}$$
$$2R_1+R_2 \le \left\{\begin{array}{l} \gamma_1 \mathsf{C}\left(\tfrac{\mathsf{SNR}_1}{1+\mathsf{INR}_2}\right) + q_{11}\mathsf{C}(\mathsf{SNR}_1+\mathsf{INR}_1) + q_{10}\mathsf{C}(\mathsf{SNR}_1) + q_{01}\mathsf{C}(\mathsf{INR}_1) \\ +q_{11}\mathsf{C}\left(\mathsf{INR}_2+\tfrac{\mathsf{SNR}_2}{1+\mathsf{INR}_1}\right) + q_{10}\mathsf{C}(\mathsf{INR}_2) + q_{01}\mathsf{C}\left(\tfrac{\mathsf{SNR}_2}{1+\mathsf{INR}_1}\right) \end{array}\right\} \tag{6}$$
$$R_1+2R_2 \le \left\{\begin{array}{l} \gamma_2 \mathsf{C}\left(\tfrac{\mathsf{SNR}_2}{1+\mathsf{INR}_1}\right) + q_{11}\mathsf{C}(\mathsf{SNR}_2+\mathsf{INR}_2) + q_{01}\mathsf{C}(\mathsf{SNR}_2) + q_{10}\mathsf{C}(\mathsf{INR}_2) \\ +q_{11}\mathsf{C}\left(\mathsf{INR}_1+\tfrac{\mathsf{SNR}_1}{1+\mathsf{INR}_2}\right) + q_{01}\mathsf{C}(\mathsf{INR}_1) + q_{10}\mathsf{C}\left(\tfrac{\mathsf{SNR}_1}{1+\mathsf{INR}_2}\right) \end{array}\right\} \tag{7}$$

$$R_1 \le \gamma_1 \mathsf{C}\left((1-\rho^2)(\mathsf{SNR}_1+\mathsf{INR}_2)\right) \tag{8}$$
$$R_1 \le q_{11}\mathsf{C}\left(\mathsf{SNR}_1+\mathsf{INR}_1+2\rho\sqrt{\mathsf{SNR}_1\cdot\mathsf{INR}_1}\right) + q_{10}\mathsf{C}(\mathsf{SNR}_1) + q_{01}\mathsf{C}(\mathsf{INR}_1) \tag{9}$$
$$R_2 \le \gamma_2 \mathsf{C}\left((1-\rho^2)(\mathsf{SNR}_2+\mathsf{INR}_1)\right) \tag{10}$$
$$R_2 \le q_{11}\mathsf{C}\left(\mathsf{SNR}_2+\mathsf{INR}_2+2\rho\sqrt{\mathsf{SNR}_2\cdot\mathsf{INR}_2}\right) + q_{10}\mathsf{C}(\mathsf{SNR}_2) + q_{01}\mathsf{C}(\mathsf{INR}_2) \tag{11}$$
$$R_1+R_2 \le \gamma_1 \mathsf{C}\left(\tfrac{(1-\rho^2)\mathsf{SNR}_1}{1+(1-\rho^2)\mathsf{INR}_2}\right) + q_{11}\mathsf{C}\left(\mathsf{SNR}_2+\mathsf{INR}_2+2\rho\sqrt{\mathsf{SNR}_2\cdot\mathsf{INR}_2}\right) + q_{01}\mathsf{C}(\mathsf{SNR}_2) + q_{10}\mathsf{C}(\mathsf{INR}_2) \tag{12}$$
$$R_1+R_2 \le \gamma_2 \mathsf{C}\left(\tfrac{(1-\rho^2)\mathsf{SNR}_2}{1+(1-\rho^2)\mathsf{INR}_1}\right) + q_{11}\mathsf{C}\left(\mathsf{SNR}_1+\mathsf{INR}_1+2\rho\sqrt{\mathsf{SNR}_1\cdot\mathsf{INR}_1}\right) + q_{10}\mathsf{C}(\mathsf{SNR}_1) + q_{01}\mathsf{C}(\mathsf{INR}_1) \tag{13}$$

II. Problem Formulation

The two-user bursty Gaussian interference channel considered in this work is depicted in Fig. 1 and defined as follows. User i has a message W_i to be reliably delivered from transmitter i (Txi) to receiver i (Rxi), $i = 1, 2$. The transmit signal at Txi is $X_i \in \mathbb{C}$, for $i = 1, 2$. For $(i, j) = (1, 2), (2, 1)$, the received signal at Rxi at time t is

$$Y_i[t] = h_{ii}\widetilde{X}_i[t] + h_{ij}\widetilde{X}_j[t] + Z_i[t],$$
$$\widetilde{X}_i[t] := S_i[t]X_i[t],\ \ \widetilde{X}_j[t] := S_j[t]X_j[t],$$

where the two independent additive noise terms $Z_1[t], Z_2[t]$ are $\mathcal{CN}(0, 1)$ and i.i.d. over time. For $i = 1, 2$, the burstiness state $S_i[t]$ that governs the activity of user i at time slot t, is a stationary and ergodic Bernoulli process, where γ_i is the probability of $S_i[t] = 1$ (user i being on). The joint process $(S_1[t], S_2[t])$ is also stationary and ergodic, and at each time slot t it follows the joint probability distribution:

$$\Pr\{S_1 = k, S_2 = l\} = q_{kl}, \quad k, l \in \{0, 1\}.$$

Therefore, $\gamma_1 = q_{10}+q_{11}$ and $\gamma_2 = q_{01}+q_{11}$ by definition. The channel inputs are subject to average unit power constraint:

$$\frac{1}{N}\sum_{t=1}^{N} |X_i[t]|^2 \le 1$$

for block length N. We denote signal-to-noise ratios $\mathsf{SNR}_i := |h_{ii}|^2$ and interference-to-noise ratios $\mathsf{INR}_i := |h_{ij}|^2$ at Rxi, for $i = 1, 2$ respectively.

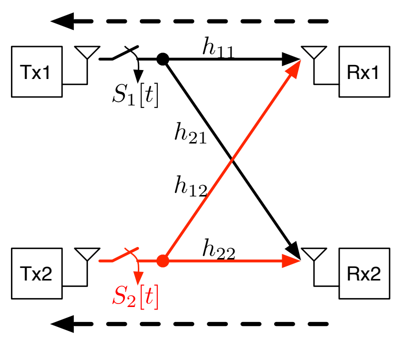

Fig. 1. Two-user Bursty Interference Channel

Three feedback models are considered in this work:

(1) *No feedback*: the transmit signal from Txi at time t, $X_i[t]$ is determined by $(W_i, S_i[1:t])$, for $i = 1, 2$.
(2) *Delayed state feedback*: Rxi only feeds S_j back to Txi, and $X_i[t]$ is determined by $(W_i, S_i[1:t])$ and $S_j[1:t-1]$ for $(i, j) = (1, 2), (2, 1)$.
(3) *Channel output feedback*: full feedback from Rxi to Txi is available, and $X_i[t]$ is determined by $(W_i, S_i[1:t])$ and $(Y_i[1:t-1], S_j[1:t-1])$ for $(i, j) = (1, 2), (2, 1)$.

Throughout this paper, $X[1:N]$ and X^N interchangeably denote the sequence $\{X[1], \cdots, X[N]\}$. For notational convenience, let $\mathsf{C}(x)$ denote $\log(1+x)$ for all $x \ge 0$. We use the short hand notation $\underline{S}$ to denote (S_1, S_2).

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III. Main Result

Our main result is summarized in the two theorems below.

Theorem 3.1 (Non-Feedback and Delayed State Feedback): For any non-negative (R_1, R_2) that satisfies (1) – (7), the rate tuple $(R_1 - \gamma_1, R_2 - \gamma_2)$ is achievable without any feedback. Conversely, if a rate tuple (R_1, R_2) is achievable when delayed state feedback is available, it satisfies (1) – (7).

Theorem 3.2 (Channel Output Feedback): For any non-negative (R_1, R_2) that satisfies (8) – (13) for some $0 \le \rho \le 1$, the rate tuple $(R_1 - \max(q_{11} + \gamma_1, \gamma_2), R_2 - \max(q_{11} + \gamma_2, \gamma_1))$ is achievable with channel output feedback. Conversely, if a rate tuple (R_1, R_2) is achievable when channel output feedback is available, it satisfies (8) – (13) for some $0 \le \rho \le 1$.

Remark 3.1: When both users are always on, that is, $\gamma_1 = \gamma_2 = q_{11} = 1$, it becomes the non-bursty IC and we recover the results in [1] and [2].

A. Benefit of Channel Output Feedback

To better compare the two results and visualize the benefit of feedback as well as the impact of burstiness in traffic, we adopt the notion of *generalized degrees of freedom* (GDoF), originally proposed in [1].

In the following, we focus on a symmetric setup, where:

(1) channel is symmetric: $\mathsf{SNR}_1 = \mathsf{SNR}_2 = \mathsf{SNR}$, $\mathsf{INR}_1 = \mathsf{INR}_2 = \mathsf{INR}$, and
(2) burstiness state statistics are symmetric: $\gamma_1 = \gamma_2 = \gamma$, that is, the (marginal) probability of user 1 being on is the same as that of user 2. We then define $p := \frac{q_{11}}{\gamma}$, which can be intuitively thought of as the probability that a receiver gets interfered when its own transmitter is on.

In this symmetric setup, for comparison, we focus on the symmetric rate point of the capacity region

$$C_{\text{sym}} := \max_{(R,R):\text{achievable}} R.$$

Extending from the definition in [1] we have the following definition of GDoF in the bursty case:

$$d_{\text{sym}}(\alpha, p) := \lim_{\mathsf{SNR}\to\infty,\, \frac{\log \mathsf{INR}}{\log \mathsf{SNR}} = \alpha} \frac{C_{\text{sym}}}{\gamma \log \mathsf{SNR}}.$$

The difference of the above definition from that in the non-bursty IC is two-fold: first, in the normalization there is a pre-log factor γ to reflect the portion of time in which each user is on for transmission; second, GDoF in the bursty case depends not only on the strength of interference (α), but also on the portion of time that a receiver is interfered (p).

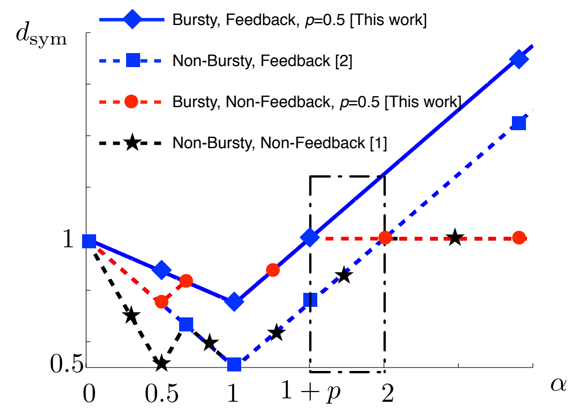
Theorem 3.1 and 3.2 directly imply the following corollary:

Corollary 3.1 (Generalized Degrees of Freedom): The non-feedback GDoF and feedback GDoF of the symmetric bursty IC are as follows:

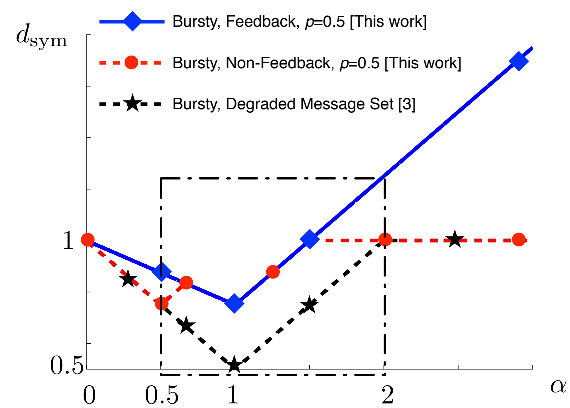
$$d_{\text{sym}}^{\text{NFB}} = \begin{cases} 1 - p\alpha, & 0 \le \alpha < 1/2) \\ 1 - p(1-\alpha), & 1/2 \le \alpha < 2/3 \\ 1 - p\alpha/2, & 2/3 \le \alpha < 1 \\ (1 - p + \alpha)/2, & 1 \le \alpha < 1 + p \\ 1, & \alpha \ge 1 + p \end{cases}$$

$$d_{\text{sym}}^{\text{FB}} = \begin{cases} 1 - p\alpha/2, & 0 \le \alpha < 1 \\ (1 - p + \alpha)/2, & \alpha \ge 1 \end{cases}$$

Compared with the non-bursty case, feedback provides improvement in GDoF in the regime $1 + p \le \alpha \le 2$ in the bursty case. Such an GDoF gain implies an unbounded capacity gain asymptotically. For an illustration, see Fig. 2(a), where we plot the GDoF versus α for non-bursty IC with and without feedback and bursty IC with and without feedback for $p = 1/2$.



(a) Benefit of Feedback



(b) Benefit of Coding over Blocks

Fig. 2. Comparison of Generalized Degrees of Freedom

B. Benefit of Coding over Blocks of Transmission

A natural strategy for the non-feedback case is the degraded-message-set approach [3], which does not exploit the opportunity to code over multiple blocks of transmission. In order to see the benefit of coding over multiple transmission blocks, we compute the achievable throughput of the degraded message set approach and the corresponding GDoF. In Fig. 2(b), we plot the GDoF achieved by the degraded message set approach and compare it to the optimal non-feedback GDoF. As shown in the plot, we see that coding over multiple blocks provides a significant increase in GDoF in the regime $1/2 \le \alpha \le 2$. Therefore, the benefit of coding over multiple transmission blocks is prominent in the moderate interference regime.

IV. Without Feedback or Delayed State Feedback

In this section we prove Theorem 3.1. For achievability we employ the coding scheme in the non-bursty case [1] [4]. For the converse part we let a genie provide non-causal state information to the transmitters, and then employ techniques used in the non-bursty case [1] to prove the outer bounds (1) – (7). Since the genie provide non-causal state information to the transmitters, these outer bounds are valid even when delayed state feedback is available.

A. Converse Proof

As mentioned above, the idea behind the converse proof is straightforward. In the following, we shall demonstrate the idea through the proof of (5) and omit the proof details of the rest of outer bounds.

We consider the case with delayed state feedback. By Fano's inequality and data processing inequality, for any achievable rate tuple (R_1, R_2), we have

$$\begin{aligned}
&N\left(R_1+R_2-\epsilon_N\right) \le I\left(W_1;Y_1^N,\underline{S}^N\right)+I\left(W_2;Y_2^N,\underline{S}^N\right)\\
&\overset{(a)}{=} I\left(W_1;Y_1^N|\underline{S}^N\right)+I\left(W_2;Y_2^N|\underline{S}^N\right)\\
&\le I\left(W_1;Y_1^N,V_1^N|\underline{S}^N\right)+I\left(W_2;Y_2^N,V_2^N|\underline{S}^N\right)\\
&= h\left(Y_1^N,V_1^N|\underline{S}^N\right)-h\left(V_2^N,Z_2^N|\underline{S}^N,W_1\right)\\
&\quad + h\left(Y_2^N,V_2^N|\underline{S}^N\right)-h\left(V_1^N,Z_1^N|\underline{S}^N,W_2\right)\\
&\overset{(b)}{=} h\left(Y_1^N,V_1^N|\underline{S}^N\right)-h\left(V_2^N|\underline{S}^N\right)-h\left(Z_2^N\right)\\
&\quad + h\left(Y_2^N,V_2^N|\underline{S}^N\right)-h\left(V_1^N|\underline{S}^N\right)-h\left(Z_1^N\right)\\
&= h\left(Y_1^N|\underline{S}^N,V_1^N\right)+h\left(Y_2^N|\underline{S}^N,V_2^N\right)-2N\log(\pi e)\\
&\le N\cdot \text{Right Hand Side of (5)},
\end{aligned}$$

where $\epsilon_N \to 0$ as $N \to \infty$. Here $V_1 := h_{21}\widetilde{X}_1 + Z_2$ and $V_2 := h_{12}\widetilde{X}_2 + Z_1$. (a) is due to the fact that messages are independent of channel states. (b) is due to the fact that under delayed state feedback, $W_1 \leftrightarrow \underline{S}^N \leftrightarrow X_2^N$ and $W_2 \leftrightarrow \underline{S}^N \leftrightarrow X_1^N$, and that noises are independent of everything else.

Hence for any achievable (R_1, R_2), (5) holds.

B. Achievability and Bounded Gap to Capacity

Directly applying the Han-Kobayashi superposition coding scheme [4] without making use of the knowledge of the delayed state feedback, we have the following achievable region (Theorem 6.4. in [5] without time-sharing):

$$R_1 \le I\left(X_1;Y_1|\underline{S},U_2\right) \tag{14}$$
$$R_2 \le I\left(X_2;Y_2|\underline{S},U_1\right) \tag{15}$$
$$R_1+R_2 \le I\left(X_1;Y_1|\underline{S},U_1,U_2\right)+I\left(X_2,U_1;Y_2|\underline{S}\right) \tag{16}$$
$$R_1+R_2 \le I\left(X_2;Y_2|\underline{S},U_1,U_2\right)+I\left(X_1,U_2;Y_1|\underline{S}\right) \tag{17}$$
$$R_1+R_2 \le I\left(X_1,U_2;Y_1|\underline{S},U_1\right)+I\left(X_2,U_1;Y_2|\underline{S},U_2\right) \tag{18}$$
$$\begin{aligned}2R_1+R_2 \le\ & I\left(X_1;Y_1|\underline{S},U_1,U_2\right)+I\left(X_1,U_2;Y_1|\underline{S}\right)\\ &+I\left(X_2,U_1;Y_2|\underline{S},U_2\right)\end{aligned} \tag{19}$$
$$\begin{aligned}R_1+2R_2 \le\ & I\left(X_2;Y_2|\underline{S},U_1,U_2\right)+I\left(X_2,U_1;Y_2|\underline{S}\right)\\ &+I\left(X_1,U_2;Y_1|\underline{S},U_1\right)\end{aligned} \tag{20}$$

for some $p\left(u_1,x_1\right)p\left(u_2,x_2\right)$. Choose the codebook generating random variables to be Gaussian distributed as follows:

$$U_i \sim \mathcal{CN}\left(0,Q_{ic}\right), X_{ip} \sim \mathcal{CN}\left(0,Q_{ip}\right), X_i = U_i + X_{ip},$$

for $i = 1, 2$. Here U_i and X_{ip} are independent random variables and $Q_{ic} + Q_{ip} = 1$. According to the intuition outlined in [1], we choose $Q_{1p} = \min\left\{1, \mathsf{INR}_2^{-1}\right\}$ and $Q_{2p} = \min\left\{1, \mathsf{INR}_1^{-1}\right\}$ so that the private part of the transmitted signals at the unintended receivers will lie at or below the noise level.

After straightforward computation, we bound the gap between the outer bounds (1) – (7) and the corresponding inner bounds (14) – (20) as follows:

$$\begin{aligned}
R_1, R_2: &\quad (1)-(14) \le q_{11},\ (2)-(15) \le q_{11}\\
R_1+R_2: &\quad (3)-(16) \le q_{11}+\gamma_1,\ (4)-(17) \le q_{11}+\gamma_2,\\
&\quad (5)-(18) \le \gamma_1+\gamma_2\\
2R_1+R_2: &\quad (6)-(19) \le q_{11}+\gamma_1+\gamma_2\\
R_1+2R_2: &\quad (7)-(20) \le q_{11}+\gamma_1+\gamma_2
\end{aligned}$$

Hence, for any non-negative (R_1, R_2) that satisfies (1) – (7), the rate tuple $(R_1 - \gamma_1, R_2 - \gamma_2)$ is achievable.

V. Channel Output Feedback

In this section we prove Theorem 3.2. Similar to the proof of Theorem 3.1, for achievability we take the scheme in the non-bursty case [2], and for the converse we again let a genie provide non-causal state information to the transmitters, and then employ techniques used in the non-bursty case [2] to prove the outer bounds (8) – (13).

A. Converse Proof

Once again, since the idea of the proof is straightforward, we omit most of the proof details. Below we develop the proof of the sum rate outer bound (12) to illustrate the idea.

By Fano's inequality and data processing inequality, for any achievable rate tuple (R_1, R_2), we have

$$\begin{aligned}
&N\left(R_1+R_2-\epsilon_N\right) \le I\left(W_1;Y_1^N|\underline{S}^N\right)+I\left(W_2;Y_2^N|\underline{S}^N\right)\\
&\le I\left(W_1;Y_1^N,Y_2^N|\underline{S}^N,W_2\right)+I\left(W_2;Y_2^N|\underline{S}^N\right)\\
&= h\left(Y_1^N,Y_2^N|\underline{S}^N,W_2\right)-h\left(Y_1^N,Y_2^N|\underline{S}^N,W_1,W_2\right)\\
&\quad + h\left(Y_2^N|\underline{S}^N\right)-h\left(Y_2^N|\underline{S}^N,W_2\right)\\
&\overset{(a)}{=} h\left(Y_1^N|\underline{S}^N,W_2,Y_2^N\right)+h\left(Y_2^N|\underline{S}^N\right)-2N\log(\pi e)\\
&\overset{(b)}{\le} h\left(Y_1^N|\underline{S}^N,V_1^N,X_2^N\right)+h\left(Y_2^N|\underline{S}^N\right)-2N\log(\pi e)\\
&\le N\cdot \text{Right Hand Side of (12)},
\end{aligned}$$

for some $0 \le \rho \le 1$, where $\epsilon_N \to 0$ as $N \to \infty$. Here ρ denote the absolute value of the correlation between X_1 and X_2 of the single-letter input distribution $p\left(X_1, X_2\right)$. (a) is due to the fact that

$$\begin{aligned}
&h\left(Y_1^N,Y_2^N|\underline{S}^N,W_1,W_2\right)\\
&= \sum_{t=1}^{N} h\left(Y_1[t],Y_2[t]|\underline{S}^N,W_1,W_2,Y_1^{t-1},Y_2^{t-1}\right)
\end{aligned}$$

$$= \sum_{t=1}^{N} h\left(Y_1[t], Y_2[t] | \underline{S}^N, W_1, W_2, Y_1^{t-1}, Y_2^{t-1}, X_1[t], X_2[t]\right)$$
$$= \sum_{t=1}^{N} h\left(Z_1[t], Z_2[t] | \underline{S}^N, W_1, W_2, Y_1^{t-1}, Y_2^{t-1}, X_1[t], X_2[t]\right)$$
$$= \sum_{t=1}^{N} h\left(Z_1[t], Z_2[t]\right) = 2N\log(\pi e)$$

(b) is due to the fact that

$$h\left(Y_1^N | \underline{S}^N, W_2, Y_2^N\right) = h\left(Y_1^N | \underline{S}^N, W_2, Y_2^N, X_2^N\right)$$
$$= h\left(Y_1^N | \underline{S}^N, W_2, V_1^N, X_2^N\right) \le h\left(Y_1^N | \underline{S}^N, V_1^N, X_2^N\right).$$

Hence for any achievable (R_1, R_2), (12) holds.

B. Achievability and Bounded Gap to Capacity

Directly applying the block Markov coding scheme in [2], we have the following achievable rate region:

$$R_1 \le I\left(X_1; Y_1 | \underline{S}, U, U_1, U_2\right) + I\left(U_1; Y_2 | \underline{S}, U, X_2\right) \quad (21)$$
$$R_1 \le I\left(U, U_2, X_1; Y_1 | \underline{S}\right) \quad (22)$$
$$R_2 \le I\left(X_2; Y_2 | \underline{S}, U, U_1, U_2\right) + I\left(U_2; Y_1 | \underline{S}, U, X_1\right) \quad (23)$$
$$R_2 \le I\left(U, U_1, X_2; Y_2 | \underline{S}\right) \quad (24)$$
$$R_1 + R_2 \le I\left(X_1; Y_1 | \underline{S}, U, U_1, U_2\right) + I\left(U, U_1, X_2; Y_2 | \underline{S}\right) \quad (25)$$
$$R_1 + R_2 \le I\left(X_2; Y_2 | \underline{S}, U, U_1, U_2\right) + I\left(U, U_2, X_1; Y_1 | \underline{S}\right) \quad (26)$$

for some $p(u)\, p(u_1, x_1 | u)\, p(u_2, x_2 | u)$. Choose the codebook generating random variables to be Gaussian distributed as follows:

$$U \sim \mathcal{CN}(0, \rho), U_i \sim \mathcal{CN}(0, Q_{ic}), X_{ip} \sim \mathcal{CN}(0, Q_{ip}),$$
$$X_i = U + U_i + X_{ip},$$

for $i = 1, 2$ and some $0 \le \rho \le 1$. Here $\{U, U_i, X_{ip}\}$ are independent random variables and $Q_{ic} + Q_{ip} = 1 - \rho$. As in the non-feedback case, we choose $Q_{1p} = \min\left\{1 - \rho, \mathsf{INR}_2^{-1}\right\}$ and $Q_{2p} = \min\left\{1 - \rho, \mathsf{INR}_1^{-1}\right\}$ so that the private part of the transmitted signals at the unintended receivers will lie at or below the noise level.

After straightforward computation, we bound the gap between the outer bounds (8) – (13) and the corresponding inner bounds (21) – (26) as follows:

R_1 :	$(8) - (21) \le q_{11} + \gamma_1,\ (9) - (22) \le \gamma_2$
R_2 :	$(10) - (23) \le q_{11} + \gamma_2,\ (11) - (24) \le \gamma_1$
$R_1 + R_2$:	$(12) - (25) \le q_{11} + \gamma_1,\ (13) - (26) \le q_{11} + \gamma_2$

Hence, for any non-negative (R_1, R_2) that satisfies (8) – (13), the rate tuple $(R_1 - \max(q_{11} + \gamma_1, \gamma_2), R_2 - \max(q_{11} + \gamma_2, \gamma_1))$ is achievable.

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