Convex Optimization for Precoder Design in MIMO Interference Networks

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***Abstract*— Optimal precoder design for weighted sum-rate maximization in multiple-input multiple-output interference networks is studied. For this well known non-convex optimization problem, convex approximations based on interference alignment are developed, for both single-beam and multi-beam cases. Precoder design methods that consist of two phases, an interference alignment phase and a post-alignment optimization phase, are proposed. The interference alignment solution is taken as the input to the post-alignment optimization phase. For post-alignment weighted sum-rate maximization, novel iterative distributed algorithms are proposed based on the developed convex approximations. Simulation results show that the proposed algorithms achieve promising weighted sum-rate gains over existing interference alignment algorithms. Interestingly, for the multi-beam case, significant gain is achieved at all SNRs, including the high SNR regime.**

I. Introduction

We investigate the problem of precoder design for maximizing the weighted sum-rate in multiple-input multiple-output (MIMO) interference channels with arbitrary constant channel coefficients. We assume that each user treats interference from other users as noise. It is well known that, due to interference coupling, the problem is a non-convex optimization and is hard to solve [1]. In the high SNR regime, there has been recent progress on maximizing the *sum degrees of freedom*, exploiting the idea of interference alignment [2]. It has been shown that maximizing the sum degrees of freedom is still an NP hard problem [3]. A closely related problem is the *feasibility* of interference alignment, namely, whether a given set of desired degrees of freedom is achievable for all the users [4]. There have been several recent theoretical breakthroughs on determining the conditions under which interference alignment is feasible [4], [5], [6], [7]. In addition, many algorithms have been developed for finding numerical solutions in any given channel realization that successfully achieve interference alignment and the desired degrees of freedom [3], [8], [9], [10], [11], [12], [13], [14], [15], [16]. All this algorithmic work is based on iterative and distributed optimization of linear precoders and receive filters. In particular, the max-SINR (signal-to-interference-plus-noise ratio) algorithm developed in the seminal work of [8] has a very favorable *sum-rate* performance at all SNRs, and is also computationally light. An alternative approach for maximizing the weighted sum-rate in MIMO interference channels is to use interference pricing [17]. However, it has been shown that interference pricing approaches are outperformed by interference alignment algorithms in the high SNR regime [14].

With a weighted sum-rate objective, when the weights among the users vary, the optimal solutions can vary significantly. While most interference alignment algorithms try to maximize sum degrees of freedom or sum-rate [8]–[13], several works have taken different priorities among the users into account via convex optimization approaches. In [3], semidefinite programming (SDP) approximations are exploited. However, it is unclear whether this approach arrives at solutions that have good interference alignment properties. Hence it is unclear whether this approach has comparable performance at high SNRs relative to other iterative interference alignment algorithms. In [14], a weighted minimum mean square error (MMSE) beamforming approach is developed for single-beam cases where each user utilizes exactly one signalling dimension. For this technique, it was shown that, while the weighted MMSE algorithms outperform interference alignment algorithms in weighted sum-rate at low and intermediate SNRs, their performance at high SNRs is very close. The weighted MMSE approach was generalized to multi-beam cases in [15] and [16].

In this paper, we consider both single-beam and multi-beam cases, and address the different user priorities by performing weighted sum-rate maximization. The optimization variables are the precoding matrices of all the users, and we assume that the number of signalling dimensions of each user is given. To maintain a first order optimality, we ensure that through interference alignment the developed solutions always achieve the expected numbers of degrees of freedom for all the users. Then, based on the particular properties of the interference alignment solutions, we make convex approximations in maximizing the weighted sum-rate objective. We propose iterative algorithms that optimize the precoding matrices and the receive filters based on distributed convex optimization. We show that, at all SNRs, (including the high SNR regime,) the proposed algorithms achieve significant improvement over the max-SINR algorithm [8] applied with a large number of random initializations in the multi-beam case.

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While convergence of the proposed algorithms is observed in all the simulated cases, the theoretical proof of this convergence remains open. The main challenge lies in analyzing the distributed nature of the algorithms in which, at each iteration, different users approximate the global objective differently.

The remainder of the paper is organized as follows. The system model is established in Section II. In Section III we develop distributed convex approximations of weighted sum-rate maximization based on the properties of interference alignment. Iterative distributed algorithms that optimize precoders and receive filters are proposed in Section IV. In Section V, the performance of the proposed algorithms are evaluated and compared with existing interference alignment algorithms. We end with a brief conclusion in Section VI.

II. System Model

We consider MIMO K-user interference channels. For user k, we denote by M_k and N_k the numbers of its transmit and receive antennas, respectively. We denote by $\boldsymbol{H}_{kj} \in \mathbb{R}^{N_k \times M_j}$ the constant real channel matrix from transmitter j to receiver k. (We note that complex channel gains and signalling can be equivalently transformed into real ones.) At receiver k,

$$\mathbf{y}_k[t] = \sum_{j=1}^{K} \mathbf{H}_{kj} \boldsymbol{x}_j[t] + \boldsymbol{z}_k[t], \tag{1}$$

where $\mathbf{x}_j[t] \in \mathbb{R}^{M_j \times 1}, \boldsymbol{y}_k[t] \in \mathbb{R}^{N_k \times 1}$ are the transmitted and the received signal vectors of user k, and $\mathbf{z}_k[t] \sim \mathcal{N}(0, n_k \mathbf{I}_k)$ is the noise vector with n_k as the noise variance at each receive antenna of user k. We focus on linear precoding schemes: For user k, let $\mathbf{V}_k \in \mathbb{R}^{M_k \times d_k}$ be its precoding matrix, and $\forall k$,

$$\mathbf{x}_k[t] = \mathbf{V}_k \mathbf{s}_k[t], \tag{2}$$

where d_k is the number of independent information streams of user k, and $\mathbf{s}_k[t] \in \mathbb{R}^{d_k \times 1}$ is the information vector of user k whose d_k elements each independently encodes one of the d_k streams of user k. We assume that each element of $\boldsymbol{s}_k[t]$ is drawn from an independently generated Gaussian codebook with unit power. For notational simplicity, we omit the time index t from now on.

A. Decoding Assumptions

We assume that each user treats signals from all the other users as noise. For user k's own d_k streams, we assume that they are *jointly* decoded at receiver k. This gives rise to the following achievable rate of user k [18]:

$$R_k = \frac{1}{2} \log\det \left(\boldsymbol{I} + \boldsymbol{V}_k^T \boldsymbol{H}_{kk}^T \boldsymbol{B}_k^{-1} \boldsymbol{H}_{kk} \boldsymbol{V}_k \right), \tag{3}$$

where

$$\boldsymbol{B}_k = \sum_{j \neq k} \boldsymbol{H}_{kj} \boldsymbol{V}_j \boldsymbol{V}_j^T \boldsymbol{H}_{kj}^T + n_k \boldsymbol{I} \tag{4}$$

is the covariance matrix of the received interference plus noise at receiver k. We now formulate the *weighted sum-rate* maximization problem as follows:

$$\max_{\boldsymbol{V}_k, k=1,\ldots,K} \sum_{k=1}^{K} w_k R_k, \qquad s.t. \;\; \mathrm{tr}(\boldsymbol{V}_k \boldsymbol{V}_k^T) \leq P_k, k = 1, \ldots, K, \tag{5}$$

where P_k is user k's power constraint.

An alternative decoding assumption is to decode each stream by treating all other streams as noise, even if the other streams are from the same desired user. Interestingly, although treating all other streams as noise is suboptimal compared to jointly decoding all the streams of the desired user, this technique has been widely adopted in iterative interference alignment algorithms. The main reason is that it is helpful in finding solutions that achieve interference alignment, (albeit not necessarily optimized for rate objectives). Finally, an additional constraint that is commonly applied in the literature is to restrict the precoding matrices to be orthogonal matrices, particularly when the objective is minimizing leakage interference instead of maximizing rates. We note that while adding this constraint does not restrict optimality in minimizing leakage interference, it can restrict optimality in maximizing rates.

B. Linear Receive Filters

Solving (5) by directly optimizing the precoding matrices is hard because the rate function (3) is non-concave in $\{\boldsymbol{V}_k, k = 1, \ldots, K\}$. Instead, we introduce linear receiver filters as *auxiliary* optimization variables: By first applying a linear receive filter $\boldsymbol{U}_k \in \mathbb{R}^{N_k \times d_k}$ at receiver k and then jointly decoding the symbols in $\boldsymbol{s}_k$ from $\boldsymbol{U}_k^T \boldsymbol{y}_k$, the achievable rate of user k becomes

$$\begin{aligned} R_k =& \frac{1}{2} \log\det \left(\boldsymbol{I} + \boldsymbol{V}_k^T \boldsymbol{H}_{kk}^T \boldsymbol{U}_k (\boldsymbol{U}_k^T \boldsymbol{B}_k \boldsymbol{U}_k)^{-1} \boldsymbol{U}_k^T \boldsymbol{H}_{kk} \boldsymbol{V}_k \right) \\ =& \frac{1}{2} \log\det \left(\boldsymbol{I} + (\boldsymbol{U}_k^T \boldsymbol{B}_k \boldsymbol{U}_k)^{-1} \boldsymbol{U}_k^T \boldsymbol{H}_{kk} \boldsymbol{V}_k \boldsymbol{V}_k^T \boldsymbol{H}_{kk}^T \boldsymbol{U}_k \right). \end{aligned} \tag{6}$$

It can be immediately seen that there is no capacity loss, i.e., (3) = (6), as long as the following *optimal* linear receive filter is applied:

$$\boldsymbol{U}_k = \boldsymbol{B}_k^{-1} \boldsymbol{H}_{kk} \boldsymbol{V}_k. \tag{7}$$

The introduction of the receive filters enables a series of approximations of the rate function (6), based on which we develop efficient algorithms for approximately solving (5) in the following sections.

III. Distributed Convex Optimization Based on Interference Alignment

In this section, we focus on optimizing the precoding matrices with fixed receive filters. We approach this non-convex optimization using distributed convex approximation motivated by the properties of interference alignment solutions. This provides the building blocks for the iterative algorithms that we will develop in Section IV.

A. The Single-beam Case

We first consider the single-beam case in which each user transmits only one stream, i.e., $d_k = 1, \forall k$. In this case, the precoding matrices and the receive filters $\{\boldsymbol{V}_k, \boldsymbol{U}_k\}$ become *vectors*, and we denote them by $\{\boldsymbol{v}_k, \boldsymbol{u}_k, k = 1, \ldots, K\}$. Accordingly, (6) is equivalent to

$$R_k = \frac{1}{2}\log\left(1 + \frac{|\boldsymbol{u}_k^T \boldsymbol{H}_{kk}\boldsymbol{v}_k|^2}{\sum_{j\neq k}|\boldsymbol{u}_k^T \boldsymbol{H}_{kj}\boldsymbol{v}_j|^2 + n_k|\boldsymbol{u}_k|^2}\right). \quad (8)$$

Given $\{\boldsymbol{v}_k\}$, the optimal $\{\boldsymbol{u}_k\}$ are computed by (7). We now investigate the optimization of $\{\boldsymbol{v}_k\}$ with a given $\{\boldsymbol{u}_k\}$. We first make a high SINR approximation for (8):
Define $\forall k, j, \boldsymbol{h}_{kj} \triangleq \boldsymbol{H}_{kj}^T \boldsymbol{u}_k$, then

$$\begin{aligned} R_k &= \frac{1}{2}\log(1 + \frac{|\boldsymbol{h}_{kk}^T \boldsymbol{v}_k|^2}{\sum_{j\neq k}|\boldsymbol{h}_{kj}^T \boldsymbol{v}_j|^2 + n_k|\boldsymbol{u}_k|^2}) \\ &\approx \frac{1}{2}\log(\frac{|\boldsymbol{h}_{kk}^T \boldsymbol{v}_k|^2}{\sum_{j\neq k}|\boldsymbol{h}_{kj}^T \boldsymbol{v}_j|^2 + n_k|\boldsymbol{u}_k|^2}), \end{aligned} \quad (9)$$

We note that the approximation gap is at most $\frac{1}{2}$ bit, provided that SINR ≥ 0 dB which is typically true with interference alignment.

For solving (5), instead of jointly optimizing $\boldsymbol{v}_1, \ldots, \boldsymbol{v}_K$, we consider *distributed* algorithms in which each user optimizes its own precoding vector while treating other users' precoding vectors as fixed. However, we let each user keep a *global perspective* by keeping the weighted sum-rate as its objective function. Specifically, for any user k, it optimizes its precoding vector $\boldsymbol{v}_k$ as follows:

$$\begin{aligned} &\max_{|\boldsymbol{v}_k|^2\leq P_k} \frac{1}{2}\sum_{j=1}^{K} w_j \log\left(\frac{|\boldsymbol{h}_{jj}^T \boldsymbol{v}_j|^2}{\sum_{i\neq j}|\boldsymbol{h}_{ji}^T \boldsymbol{v}_i|^2 + n_j|\boldsymbol{u}_j|^2}\right) \\ \Leftrightarrow &\max_{|\boldsymbol{v}_k|^2\leq P_k}\left(w_k \log|\boldsymbol{h}_{kk}^T \boldsymbol{v}_k|^2 - \sum_{j\neq k} w_j \log\left(|\boldsymbol{h}_{jk}^T \boldsymbol{v}_k|^2 + N_j\right)\right), \end{aligned} \quad (10)$$

where $\quad N_j \triangleq \sum_{i\neq j, i\neq k} |\boldsymbol{h}_{ji}^T \boldsymbol{v}_i|^2 + n_j|\boldsymbol{u}_j|^2$

is the *aggregate of noise and leakage interference* at receiver $j(\neq k)$ from users *other than* user k, and the notation $\Leftrightarrow$ denotes equivalence. By solving (10), user k takes into account both its own desired signal and its interference to the other $K-1$ users.

Problem (10) remains a non-convex optimization which is hard to solve. To develop an accurate convex approximation, we exploit the intuition from *interference alignment*. Firstly,

$$\begin{aligned} (10) \Leftrightarrow &\max_{|\boldsymbol{v}_k|^2\leq P_k}\left(2w_k \log|\boldsymbol{h}_{kk}^T \boldsymbol{v}_k| - \sum_{j\neq k} w_j \log\left(1 + \frac{|\boldsymbol{h}_{jk}^T \boldsymbol{v}_k|^2}{N_j}\right)\right) \\ &- \sum_{j\neq k} w_j \log(N_j). \end{aligned} \quad (11)$$

Note that $|\boldsymbol{h}_{jk}^T \boldsymbol{v}_k|^2 + N_j$ constitutes the total leakage interference plus noise at receiver j: $|\boldsymbol{h}_{jk}^T \boldsymbol{v}_k|^2$ is the leakage interference solely from user k, while N_j is the leakage interference from all the other $K-2$ interferers ($\{1, \ldots, K\}\backslash j\backslash k$) plus noise. Now, we analyze the consequence of successful interference alignment as follows:

- If near perfect interference alignment is achieved, the total leakage interference will be less than or comparable to the noise level.
- Even if the leakage interference significantly exceeds the noise level, with sufficient interference alignment, the interference from user k shall be aligned to mostly lie in the subspace spanned by the interference from users $\{1, \ldots, K\}\backslash j\backslash k$.

Thus, with sufficient interference alignment, $|\boldsymbol{h}_{jk}^T \boldsymbol{v}_k|^2/N_j$ will be relatively small. This motivates us to make the following approximation:

$$\log\left(1 + \frac{|\boldsymbol{h}_{jk}^T \boldsymbol{v}_k|^2}{N_j}\right) \approx \frac{|\boldsymbol{h}_{jk}^T \boldsymbol{v}_k|^2}{N_j}. \quad (12)$$

Accordingly, (11) is approximated as

$$\max_{|\boldsymbol{v}_k|^2\leq P_k} 2w_k \log|\boldsymbol{h}_{kk}^T \boldsymbol{v}_k| - \sum_{j\neq k} w_j \frac{|\boldsymbol{h}_{jk}^T \boldsymbol{v}_k|^2}{N_j}. \quad (13)$$

Finally, it is straightforward to see that restricting (13) to the halfspace of $\boldsymbol{h}_{kk}^T \boldsymbol{v}_k > 0$ gives the same optimal value as restricting (13) to the other halfspace $\boldsymbol{h}_{kk}^T \boldsymbol{v}_k < 0$. Therefore, (13) is equivalent to the following *convex optimization* which can be solved efficiently [19]:

$$\max_{|\boldsymbol{v}_k|^2\leq P_k, \boldsymbol{h}_{kk}^T \boldsymbol{v}_k > 0} 2w_k \log\left(\boldsymbol{h}_{kk}^T \boldsymbol{v}_k\right) - \sum_{j\neq k} w_j \frac{|\boldsymbol{h}_{jk}^T \boldsymbol{v}_k|^2}{N_j}. \quad (14)$$

B. The Multi-beam Case

We now generalize the above single-beam results to the multi-beam case, in which each user can transmit any given number of streams. Given $\{\boldsymbol{V}_k\}$, the optimal $\{\boldsymbol{U}_k\}$ are computed by (7). We now investigate the optimization of $\{\boldsymbol{V}_k\}$ with a given $\{\boldsymbol{U}_k\}$. With a high SINR approximation of (6),

$$R_k \approx \frac{1}{2}\log\det\left(\boldsymbol{U}_k^T \boldsymbol{H}_{kk}\boldsymbol{V}_k\boldsymbol{V}_k^T\boldsymbol{H}_{kk}^T\boldsymbol{U}_k\right) - \log\det\left(\boldsymbol{U}_k^T\boldsymbol{B}_k\boldsymbol{U}_k\right) \quad (15)$$

Similarly to the single-beam case, we consider distributed algorithms in which each user optimizes its own precoding matrix while treating the other users' precoding matrices as fixed, with the weighted sum-rate as the objective function. User k thus optimizes $\boldsymbol{V}_k$ as follows:

$$\begin{aligned} &\max_{\mathrm{tr}(\boldsymbol{V}_k\boldsymbol{V}_k^T)\leq P_k} \sum_{j=1}^{K} w_j R_j \quad \Leftrightarrow \\ &\max_{\mathrm{tr}(\boldsymbol{V}_k\boldsymbol{V}_k^T)\leq P_k} w_k \log\det(\tilde{\boldsymbol{H}}_{kk}^T\boldsymbol{V}_k\boldsymbol{V}_k^T\tilde{\boldsymbol{H}}_{kk}) \\ &\qquad - \sum_{j\neq k} w_j \log\det(\tilde{\boldsymbol{H}}_{jk}^T\boldsymbol{V}_k\boldsymbol{V}_k^T\tilde{\boldsymbol{H}}_{jk} + \boldsymbol{N}_j) \end{aligned} \quad (16)$$

where $\tilde{\boldsymbol{H}}_{jk} \triangleq \boldsymbol{H}_{jk}^T \boldsymbol{U}_j$, and $\boldsymbol{N}_j \triangleq \sum_{i\neq j, i\neq k} \tilde{\boldsymbol{H}}_{ji}^T \boldsymbol{V}_i \boldsymbol{V}_i^T \tilde{\boldsymbol{H}}_{ji} + n_j \boldsymbol{U}_j^T \boldsymbol{U}_j$ is the aggregate of noise and leakage interference at receiver $j(\neq k)$ from users *other than* user k. We note that (16) is a non-convex optimization, and is thus hard to solve.

Next, we develop a convex approximation of (16) which generalizes the one that we developed for the single-beam case. We define $\boldsymbol{W}_k \triangleq \boldsymbol{V}_k \boldsymbol{V}_k^T$, and rewrite (16) as

$$\max_{\boldsymbol{W}_k \in \boldsymbol{S}_+^{M_k},\ \mathrm{tr}(\boldsymbol{W}_k)\leq P_k} \Bigg(w_k \log\det(\tilde{\boldsymbol{H}}_{kk}^T \boldsymbol{W}_k \tilde{\boldsymbol{H}}_{kk}) - \sum_{j\neq k} w_j \log\det(\boldsymbol{I} + \boldsymbol{N}_j^{-1} \tilde{\boldsymbol{H}}_{jk}^T \boldsymbol{W}_k \tilde{\boldsymbol{H}}_{jk}) \Bigg) - \sum_{j\neq k} w_j \log\det(\boldsymbol{N}_j), \tag{17}$$

$$s.t. \qquad \mathrm{rank}(\boldsymbol{W}_k) = d_k, \tag{18}$$

where $\boldsymbol{S}_+^{M_k}$ is the positive semidefinite cone.

At receiver $j(\neq k)$, $\tilde{\boldsymbol{H}}_{jk}^T \boldsymbol{W}_k \tilde{\boldsymbol{H}}_{jk}$ is the leakage interference solely from user k, while $\boldsymbol{N}_j$ is the leakage interference from all the other $K-2$ interferers ($\{1,\ldots,K\}\backslash j \backslash k$) plus noise. Similar to the intuition in the single-beam case, i) when sufficient interference alignment is achieved, the interference from user k mostly lies in the subspace spanned by the interference from users $\{1,\ldots,K\}\backslash j \backslash k$, and ii) when the achieved interference is near perfect, the total leakage interference will be comparable to the noise level. Thus, with sufficient interference alignment, the largest singular value of $\boldsymbol{N}_j^{-1} \tilde{\boldsymbol{H}}_{jk}^T \boldsymbol{W}_k \tilde{\boldsymbol{H}}_{jk}$ will be relatively small, and we have the following approximation [19]:

$$\log\det\left(\boldsymbol{I} + \boldsymbol{N}_j^{-1} \tilde{\boldsymbol{H}}_{jk}^T \boldsymbol{W}_k \tilde{\boldsymbol{H}}_{jk}\right) \approx \mathrm{tr}\left(\boldsymbol{N}_j^{-1} \tilde{\boldsymbol{H}}_{jk}^T \boldsymbol{W}_k \tilde{\boldsymbol{H}}_{jk}\right). \tag{19}$$

Then, by dropping the rank constraint (18), we arrive at the following *convex relaxation* which can be solved efficiently [19]:

$$\max_{\boldsymbol{W}_k \in \boldsymbol{S}_+^{M_k},\ \mathrm{tr}(\boldsymbol{W}_k)\leq P_k} \Bigg(w_k \log\det(\tilde{\boldsymbol{H}}_{kk}^T \boldsymbol{W}_k \tilde{\boldsymbol{H}}_{kk}) - \sum_{j\neq k} w_j \mathrm{tr}\left(\boldsymbol{N}_j^{-1} \tilde{\boldsymbol{H}}_{jk}^T \boldsymbol{W}_k \tilde{\boldsymbol{H}}_{jk}\right) \Bigg). \tag{20}$$

From a relaxed solution $\boldsymbol{W}_k$, we find its singular value decomposition $\boldsymbol{W}_k = \boldsymbol{P}\boldsymbol{\Sigma}\boldsymbol{P}^T$, and obtain a rank-$d_k$ approximation by keeping the largest d_k singular values while zeroing the rest. Finally, we recover $\boldsymbol{V}_k$ by

$$\boldsymbol{V}_k = \left[\boldsymbol{p}_1 \boldsymbol{p}_2 \ldots \boldsymbol{p}_{d_k}\right] \mathrm{diag}(\sigma_1^{\frac{1}{2}}, \sigma_2^{\frac{1}{2}}, \ldots, \sigma_{d_k}^{\frac{1}{2}}), \tag{21}$$

where σ_i is the i^{th} largest singular value of $\boldsymbol{W}_k$, and $\boldsymbol{p}_i$ is its associated singular vector. In our numerical simulations, we observed that the above relaxation does not seem to be restrictive as the relaxation always returns a solution $\boldsymbol{W}_k$ with the desired rank d_k. The intuition is that, *after interference alignment is achieved* for all the users, the number of interference-free dimensions at receiver k equals d_k, and thus user k will not benefit form utilizing more than d_k dimensions. In other words, even without a rank constraint (18), the optimal $\boldsymbol{W}_k$ will most likely be rank d_k.

IV. Iterative Distributed Algorithms

In this section, we develop iterative distributed algorithms that alternatingly optimize the precoding matrices and the receive filters. We have shown that the approximations made for obtaining the convex optimization (14) and (20) are based on the achievement of interference alignment. Accordingly, we employ two phases in our algorithms: an *interference alignment phase*, and a *post-alignment optimization phase*. The purpose of the interference alignment phase is to achieve an interference alignment solution, not necessarily addressing the weighted sum-rate objective. The output solution of the alignment phase then serves as the starting point for the post-alignment weighted sum-rate maximization phase.

For the interference alignment phase, we apply the max-SINR algorithm [8] with multiple random initializations. It has been noted that the solution to which the max-SINR algorithm converges is much dependent on its initialization step [11]. To address this issue, we randomly generate a sufficiently large number of initialization steps. We perform the max-SINR algorithm with all these initial points, and select the solution that achieves the best sum-rate as the output of the interference alignment phase. In our simulations, the selected solution from this first phase always achieves interference alignment, and hence satisfies the preconditions for the convex approximations developed in Section III.

We note that multiple random initializations have also been used in many other algorithms [11], [12], [15], [16] in order to obtain a robust performance. There is clearly a trade-off between the number of initializations used and the robustness and optimality of the algorithms. In real-time applications, using a large number of random initializations may not be desirable as computation time is a major concern. To simply avoid highly sub-optimal solutions, a fewer number of random initializations may be sufficient to provide a faster solution with adequate performance. Another way of reducing the computation time is to just try a few iterations while testing different initializations, and proceed to a larger number of iterations only after the best initialization is selected. Moreover, there may be alternative methods (e.g. genetic algorithms and simulated annealing) that are able to improve solutions' optimality faster than using random initializations. These alternative methods are not addressed in this paper and are left for future work.

For the post-alignment optimization phase, we develop algorithms in the following that alternatingly optimize the precoding matrices and the receive filters, for single-beam cases and multi-beam cases, respectively (cf. Table I).

In Stage 2 of each iteration, given the current receive filters, each user optimizes its own precoding matrix in a distributed manner assuming the others' precoding matrices are fixed. Clearly, in a single Stage 2, such distributed optimization can be applied for multiple rounds in which the users update each other on the newly computed precoding

TABLE I

PROPOSED ALGORITHMS FOR POST ALIGNMENT WEIGHTED SUM-RATE MAXIMIZATION.

Algorithm 1: Iterative Distributed Convex Optimization, the Single-Beam Case

Initialize $\{\boldsymbol{v}_k\}$ from the interference alignment phase.
Repeat
Stage 1: For each $\boldsymbol{u}_k, k = 1, \ldots, K$, optimize it by solving (7) given the current $\{\boldsymbol{v}_j, j = 1, \ldots, K\}$.
Stage 2: For each $\boldsymbol{v}_k, k = 1, \ldots, K$, optimize it by solving (14) given the current $\{\boldsymbol{u}_j, j = 1, \ldots, K\}$ and $\{\boldsymbol{v}_j, j \neq k\}$,
Until approximate convergence.

Algorithm 2: Iterative Distributed Convex Optimization, the Multi-Beam Case

Initialize $\{\boldsymbol{V}_k\}$ from the interference alignment phase.
Repeat
Stage 1: For each $\boldsymbol{U}_k, k = 1, \ldots, K$, optimize it by solving (7) given the current $\{\boldsymbol{V}_j, j = 1, \ldots, K\}$.
Stage 2: For each $\boldsymbol{V}_k, k = 1, \ldots, K$, first solve $\boldsymbol{W}_k$ from (20) given the current $\{\boldsymbol{U}_j, j = 1, \ldots, K\}$ and $\{\boldsymbol{V}_j, j \neq k\}$, and then recover $\boldsymbol{V}_k$ by (21).
Until approximate convergence.

matrices after each round. However, we observed that *one* round of distributed optimization in Stage 2 of each iteration is sufficient in practice, as the rate gain from performing more rounds is negligible.

There have been a few challenges in proving the convergence of Algorithm 1 and 2 and the proof remains open. The main issue is that, while each user tries to maximize the weighted sum-rate of all the users (cf. (10), (16)), it makes an approximation (cf. (12), (19)) that is *different* from other users. A common objective that is optimized by every user after each step is yet to be characterized in this distributed optimization. However, from our simulation results, convergence of the proposed algorithms is observed in all the simulated cases, as demonstrated in Figure 1 in the following section.

V. SIMULATION RESULTS

We evaluate the performance of the proposed algorithms in 5-user interference channels. We set the weights to be $w_1 = w_2 = 0.1, w_3 = w_4 = 1, w_5 = 10$ to represent different priorities of the users. We examine Algorithm 1 in $(3\times 3, 1)^5$ interference channels, i.e., $\forall k, M_k = N_k = 3, d_k = 1$, and examine Algorithm 2 in $(6 \times 6, 2)^5$ interference channels, i.e., $\forall k, M_k = N_k = 6, d_k = 2$. We note that, in both cases, the following inequality whose satisfaction is necessary and sufficient for the existence of feasible interference alignment solutions is satisfied with equality [20]:

$$d \leq \frac{M+N}{K+1}.$$

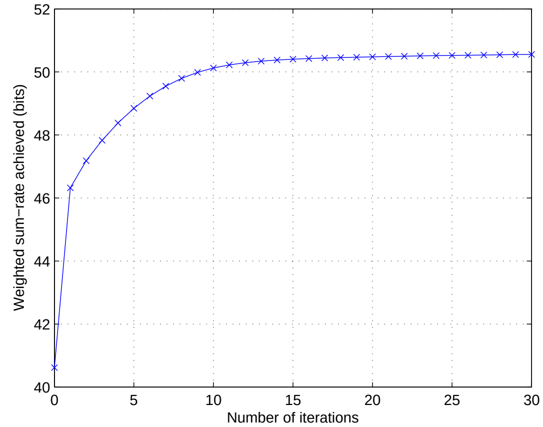

Fig. 1. Convergence behavior of Algorithm 2 for a $(6\times 6, 2)^5$ interference channel, $w_1 = w_2 = 0.1, w_3 = w_4 = 1, w_5 = 10$, SNR $= 5$ dB.

We generate all the channel matrices with independent and identically distributed $\mathcal{N}(0, 1)$ entries. For each set of channel realizations, we sweep the SNR range from 0 dB to 40 dB, and evaluate the achieved weighted sum-rate. For both cases, performance is averaged over 50 sets of random channel realizations.

We perform the interference alignment phase as follows:

1) Randomly generate 100 feasible solutions $\{\boldsymbol{V}_k, k = 1, \ldots, K\}$ as different initialization steps.
2) With each initialization step, run the max-SINR algorithm [8] for 500 iterations.
3) From the 100 final solutions, select the one that achieves the highest sum-rate as the initial point of the post-alignment optimization phase.

For the post-alignment optimization phase, we run Algorithm 1 and Algorithm 2 for the single-beam and multi-beam cases respectively, each with 30 iterations. The convergence behavior is illustrated in Figure 1 for a $(6\times 6, 2)^5$ interference channel with SNR $= 5$ dB.

We plot the simulation results on the achieved weighted sum-rate as a function of SNR for the above two cases in Figure 2 and Figure 3 respectively. The performance of the proposed algorithms is compared with that of the max-SINR algorithm applied *with 100 random initializations*.

We make the following observations:

- In the single-beam case, Algorithm 1 provides gains in the weighted sum-rate over the max-SINR algorithm applied with 100 random initializations, particularly at low and intermediate SNRs. The gain decreases as SNR increases.
- In the multi-beam case, Algorithm 2 provides significant gains in the weighted sum-rate over the max-SINR algorithm applied with 100 random initializations at all SNRs.

In particular, we note from Figure 3 that significant weighted sum-rate improvement can be made over the max-SINR algorithm with 100 random initializations even in the high SNR regime.

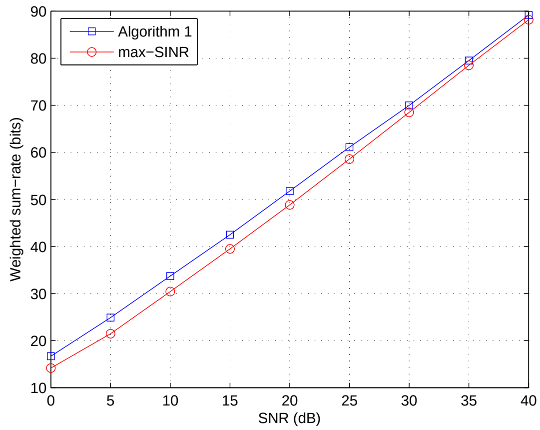

Fig. 2. Performance of Algorithm 1 in $(3 \times 3, 1)^5$ interference channels: $w_1 = w_2 = 0.1, w_3 = w_4 = 1, w_5 = 10$.

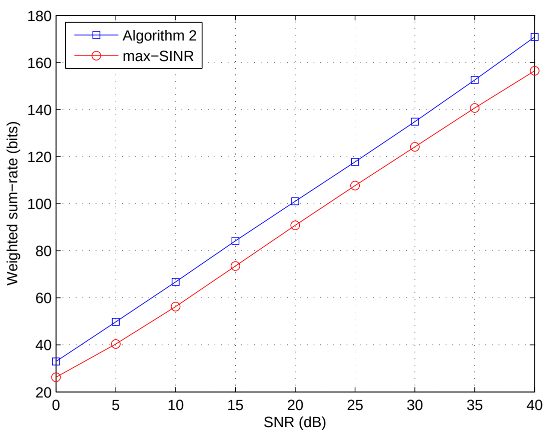

Fig. 3. Performance of Algorithm 2 in $(6 \times 6, 2)^5$ interference channels: $w_1 = w_2 = 0.1, w_3 = w_4 = 1, w_5 = 10$.

Note that while the max-SINR algorithm with a single random initialization may not have robust performance, with a sufficiently large number of random initializations, it has been demonstrated to provide a very favorable sum-rate performance that represents the state of the art of interference alignment algorithms [11]. For the single beam case, weighted sum-rate gains similar to ours over the max-SINR algorithm were also demonstrated in [14], with the gains decreasing in the high SNR regime. For the multi-beam case, however, compared to the max-SINR algorithm with a large number of random initializations, the significant weighted sum-rate gains achieved by Algorithm 2 at all SNRs have not been observed in existing algorithms reported in the literature.

VI. Conclusions

We have studied the problem of optimal precoder design for weighted sum-rate maximization in MIMO interference channels. Exploiting interference alignment, we developed convex approximations of this non-convex optimization, for both single-beam and multi-beam cases. Our precoder design methods consisted of two phases: an interference alignment phase and a post-alignment optimization phase. For the interference alignment phase, we employed the max-SINR algorithm with a sufficiently large number of random initializations to optimize the initial stage. The solution of the interference alignment phase then achieved interference alignment with high probability, and satisfied the preconditions for the proposed convex approximations. For the post-alignment optimization phase, taking the solution of the interference alignment phase as the input, we proposed novel iterative distributed algorithms based on the convex approximations. Simulation results showed that the proposed algorithms achieve promising weighted sum-rate gains over the max-SINR algorithm applied with a large number of random initializations. In the single-beam case, the achieved gain decreases as SNR increases. In the multi-beam case, however, the achieved gain is significant at all SNRs, including the high SNR regime. Convergence and approximation analysis of the proposed algorithms are topics of interest for future work.

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