2012 IEEE International Symposium on Information Theory Proceedings

Dynamic QMF for Half-Duplex Relay Networks

Ayfer Özgür
Stanford University
aozgur@stanford.edu

Suhas Diggavi
UCLA
suhas@ee.ucla.edu

***Abstract*—The value of relay nodes to enhance the error performance versus rate trade-off in wireless networks has been studied extensively. However, wireless nodes currently are constrained to only transmit or receive at a given frequency, *i.e.,* half-duplex constraint. The diversity-multiplexing tradeoff (DMT) for half-duplex networks are less understood. In the special cases where the DMT is currently known, such as the relay channel and the line network, it is achieved by either dynamic decoding or a quantize-map-forward (QMF) strategy with a fixed half-duplex schedule. The main question we investigate in this paper is whether these two strategies are sufficient to achieve the DMT of half-duplex wireless networks or we need new strategies for general setups. We propose a generalization of the two existing schemes through a dynamic QMF strategy and show that in a parallel relay channel it outperforms both earlier schemes. We also establish the DMT for the relay channel with multiple relays and multiple antennas in some special cases.**

I. INTRODUCTION

The diversity-multiplexing trade-off (DMT) [2] captures the inherent tension between rate and reliability over fading channels. It has been used to demonstrate the value of relays in wireless networks [3], [4], [5], [6]. The two critical issues that complicate the problem in relay networks is who knows what channel state and whether nodes can listen and transmit at the same time (*i.e.,* half or full duplex). The DMT of full-duplex wireless networks can be fully characterized, even with only receiver channel knowledge, which can be forwarded to the destination. For example, it can be achieved by a quantize-map-and-forward (QMF) strategy introduced in [1]. This is a simple consequence of the fact that QMF achieves the capacity of wireless relay networks within a constant gap without requiring (transmit) channel state information (CSI) at the relays [1].

In current wireless systems, nodes operate in a half-duplex mode, *i.e.,* they can not simultaneously transmit and receive signals on the same frequency band. Designing DMT optimal strategies for half-duplex networks is more challenging as it also involves an optimization over the listen and transmit times for the relays. In a fading environment where transmit CSI is unavailable at the nodes, such a listen-transmit schedule needs to be either fixed or depend only on *local* receive CSI. Characterizing the DMT of general half-duplex relay networks remains as an open problem, despite interesting progress in some special cases (see [7], [9] and references therein).

The work of A. Özgür was partially supported by the ERC grant NOWIRE ERC-2009-StG-240317. The work of S. N. Diggavi was partially supported through NSF-CPS award 1136174.

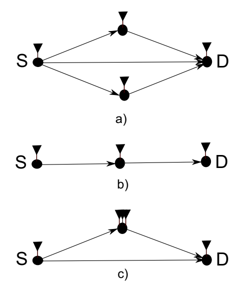

Fig. 1. a) Single relay network. b) Line relay network. c) The relay channel with multiple antennas

This work is particularly motivated by three recent results that characterize the DMT in three basic relaying setups:

- [10] shows that the optimal DMT for the relay channel with N non-interfering relays (see Figure 1-(a)) is achieved by the QMF scheme with a *fixed RX-TX* schedule for the half-duplex relays that does not depend on the channel realizations. The performance meets the full-duplex DMT.
- [12] shows that the DMT for the line relay network (see Figure 1-(b)) is achieved by dynamic decode-and-forward (DDF) [7]. In DDF the relay listens until it gathers enough mutual information to decode the transmitted message so its RX time is dynamically determined as a function of its backward channel realization and the targeted rate. The optimal performance does not reach the full-duplex DMT. The result generalizes to multiple hops and multiple transmit and receive antennas at the realays.
- [13] shows that to achieve the DMT of a single relay channel with multiple antennas at the relay node (see Figure 1-(c)) we need both of the above strategies. In this case there is a regime where DDF achieves the optimal DMT when fixed schedules are sub-optimal, and there is a second regime where a fixed schedule with QMF achieves the optimal DMT when DDF is strictly suboptimal. However, the optimal DMT does not necessarily meet the full-duplex DMT anymore.[1].

It can be show that in all other scenarios [8], [9], [11] where

[1]This result was independently derived by the authors and was part of the initially submitted version of this paper. After [13] was brought to our attention by the reviewers, we have removed this part from the final version of the paper. A subtlety regarding this result is discussed in Section III

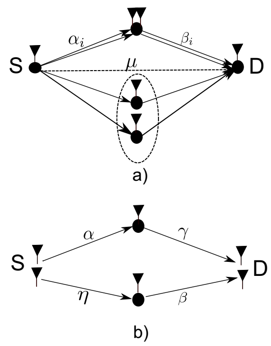

Fig. 2. a. The relay channel with multiple relays with multiple antennas studied in Section IV. b. The parallel relay network studied in Section V.

the optimal DMT is achieved by another strategy, it can be also achieved by one of two strategies above, moreover with the added benefit of avoiding extra requirements such as transmit CSI at the relay as in [8]. Therefore, the current results in the literature exhibit the following dichotomy: in all cases where the DMT of half-duplex relay networks is known it is either achieved by DDF where the relay waits until it can fully decode the source message, or by QMF with a fixed schedule independent of the channel realizations. A natural generalization of these two strategies is dynamic QMF where a relay listens for a fraction of time determined by its receive CSI that is not necessarily long enough to allow decoding of the transmitted message. The relay then quantizes maps and forwards the received signal as in the original QMF [1]. The goal of this paper is to understand whether this additional flexibility for the dynamic schedule is critical.

To make progress on this question, we first consider a natural generalization of the setups in [10] and [13], the relay channel with N non-interfering multiple-antenna relays. We are able to characterize the DMT of this setup for a class of antenna configurations. However, the optimal DMT is again achieved by QMF with fixed schedules. We then turn to a configuration of two parallel relays and demonstrate that DQMF outperforms both earlier schemes in this case. This example establishes the necessity of DQMF for achieving the DMT of general wireless networks.

II. System Model

We consider a wireless network where a source S and a destination D want to communicate with the help of half-duplex relays. In this paper we focus on two simple configurations depicted in Figure 2-(a) and (b) and studied in Sections IV and V respectively. In the first case, the communication from a single-antenna source to a single-antenna destination is assisted by multiple relays with multiple antennas. The source signal is broadcasted to the relays and the relay signals superpose at the destination. A direct link between the source and the destination may or may not be present. Figure 2-(a) shows a special case with one two-antenna relay and two single-antenna relays. In the second setup, the source communicates to the destination through two independent line networks. As opposed to the first setup, here there is no broadcast at the source or superposition at the destination. The two relays can be thought of as operating at two different frequencies. Each of the two relays is equipped with a single antenna in this case.

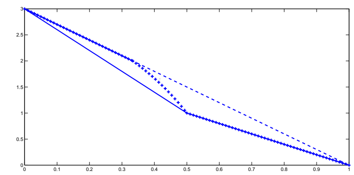

Fig. 3. '–' QMF with half-Rx half-Tx schedule for the relay, '- -' full-duplex upper bound, '+' optimal DMT.

All channels are assumed to be flat-fading, i.e. the channel gains for every link depicted in Figure 2 (from a transmit to a receive antenna) are i.i.d. circularly-symmetric complex Gaussian random variables $\mathcal{CN}(0,1)$. We assume quasi-static fading, i.e. the channel gains remain constant over the duration of the codeword and change independently from one codeword to another. The channel realizations are known at the receivers but not at the transmitters. The additive noise at the receivers is $\mathcal{CN}(0,1)$. All nodes in the system are subject to the same average power constraint specified by the average signal to noise power ratio SNR.

A scheme, consisting of a family of codes $\{\mathcal{C}_{\text{SNR}}\}$ indexed by SNR with rate $R(\text{SNR})$ and average error probability $P_e(\text{SNR})$ is said to achieve a multiplexing gain r and diversity gain d if [2]

$$\lim_{\text{SNR}\to\infty}\frac{R(\text{SNR})}{\log \text{SNR}} = r, \quad \lim_{\text{SNR}\to\infty}\frac{P_e(\text{SNR})}{\log \text{SNR}} = -d.$$

We consider codes with sufficiently long codewords so that the error event is dominated by outage, the event that the capacity of the system falls below the targeted rate. For each r, the supremum $d(r)$ of the diversity gain achievable over all families of codes is called the DMT of the system.

III. Relay with Multiple Antennas

In this section, we state the DMT of the half-duplex relay channel in Figure 1-(c).

Theorem 3.1: The DMT for the half-duplex relay channel with n transmit and receive antennas at the relay in Figure 1-(c) is given by

$$d(r) = \begin{cases} (n+1)(1-r) & 0 \le r < \frac{1}{(n+1)} \\ (n+1) - \frac{nr}{1-r} & \frac{1}{(n+1)} \le r < \frac{1}{2} \\ 2(1-r) & \frac{1}{2} \le r \le 1. \end{cases}$$

When $0 \le r \le 1/2$, the optimal DMT is achieved by dynamic decode and forward (DDF) at the relay. When $1/2 < r \le 1$, the DMT optimal strategy is quantize-map-and-forward (QMF) at the relay with a fixed half RX-half TX schedule. The full-duplex performance is reached only for $0 \le r < 1/(n+1)$. See Figure 3 for the case of two antennas at the relay.

This result was part of our ISIT submission, but reviewers pointed out [13] which had obtained this result. The only

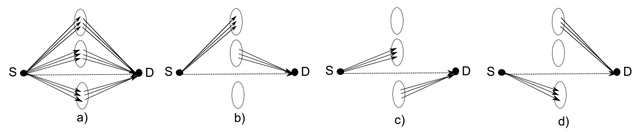

Fig. 4. The optimal fixed schedule when relays are grouped in three sets as shown in a). Each of the three states in b,c,d are active a fraction $1/3$ of the total time.

subtle difference in our approach is that the upper bounds on the DMT tradeoff differ from [13] since they are based on the general upper bound on the capacity of arbitrary half-duplex relay networks derived in [14, Section VI]. The upper bound in [14, Section VI] allows for both random (message dependent) schedules and for optimization of the transmit powers across different transmit and receive states of the network. This covers possible message dependent schedules that a simpler time schedule based upper bound does not cover. However, it then shows that the gain from such techniques is bounded by a constant independent of SNR, and hence can not impact the DMT tradeoff.

IV. Multiple Relays with Multiple Antennas

In this section, we characterize the DMT of the relay channel with N relays each with n_i transmit and receive antennas for $i = 1, \ldots, N$ when antenna configurations satisfy a specific property. The result is summarized in the following theorem.

Theorem 4.1: Let n_i, $i = 1, \ldots, N$ be such that the relay nodes can be grouped into k sets, each set containing the same total number of antennas (equal to $\sum_i n_i/k$) . Then a QMF strategy with a fixed schedule where every set receives a fraction $1/k$ of the total time and transmits a fraction $1/k$ achieves the full-duplex DMT. The full-duplex DMT is given by $d(r) = (1-r)\sum_i n_i$ when there is no direct link between the source and the destination and $d(r) = (1-r)(1+\sum_i n_i)$ when the direct link is present.

Figure 2-(a) illustrates one such configuration. Note that by grouping the two single-antenna relays together, we obtain two sets of relays each with a total number of 2 antennas. The optimal fixed schedule is to have the multiple-antenna relay listen half the time to the source node when the two single antenna relays are simultaneously transmitting to the destination. In the remaining half, the multiple-antenna relay is transmitting to the destination while the single-antenna relays are listening. Figure 4 illustrates the corresponding fixed schedule when $k = 3$. Note that if the multiple antenna relay in Figure 2-(a) contained three antennas instead of two, the network would fail to satisfy the condition in Theorem 4.1. Another configuration that fails to satisfy the condition is the relay channel with multiple antennas in Figure 1-(c) for which we know that QMF with fixed schedules is not optimal in certain regimes and the optimal DMT does not meet the full-duplex DMT (see Section III and [13]).

Proof of Theorem 4.1: We consider the case where the direct link between the source and the destination is not present. Let us enumerate the antennas at the relays as $i = 1, \ldots, m$ where $m = \sum_i n_i$ and let h_{si} and h_{id} be the channel gains between the antenna i and the source and the destination nodes respectively. Let $K_1, \ldots, K_k$ be the k sets of relays each containing an equal total number of antennas. The performance of the QMF strategy in half-duplex networks under a fixed RX-TX schedule for the relays is lower bounded in [1, Section VIII-C]. For the schedule in Figure 4, where each set $K_1, \ldots, K_k$ listens and transmits a fraction $1/k$ of the total time such that there is a single set that listens and a single set that transmits at any given time, the performance of QMF is lower bounded by $R_{QMF} \geq C_{h.d.} - \kappa$ where κ is a constant independent of SNR and

$$C_{h.d.} = \min_{\Lambda} \frac{1}{k} \sum_{j=1}^{k} \log\big(1 + \sum_{i \in \Lambda \cap K_j} |h_{id}|^2 \mathrm{SNR}\big) + \log\big(1 + \sum_{i \in \bar{\Lambda} \cap K_j} |h_{si}|^2 \mathrm{SNR}\big)$$

and the minimization is over all possible subsets Λ of the relay nodes. In the high-SNR limit, the outage event $\{R_{QMF} \leq r \log \mathrm{SNR}\}$ is equivalent to

$$\mathcal{O}(r) = \Big\{ \min_{\Lambda} \frac{1}{k} \sum_{j=1}^{k} \max_{i \in \Lambda \cap K_j} \beta_i + \max_{i \in \bar{\Lambda} \cap K_j} \alpha_i \leq r \Big\}$$

where $\alpha_i := \lim_{\mathrm{SNR} \to \infty} \frac{\log(1+|h_{si}|^2 \mathrm{SNR})}{\log \mathrm{SNR}}$, is the exponential order of $|h_{si}|^2$ or the multiplexing gain of the corresponding channel, and β_i is the exponential order of $|h_{id}|^2$. The outage probability is given by

$$P_{\mathcal{O}(r)} \doteq \mathrm{SNR}^{-d_{QMF}(r)}$$

where $\doteq$ implies that $\frac{\log P_{\mathcal{O}(r)}}{\log \mathrm{SNR}} \to -d_{QMF}(r)$ asymptotically in SNR and from [2],

$$d_{QMF}(r) = \min \qquad 2m - \sum_{i=1}^{m} \alpha_i - \sum_{i=1}^{m} \beta_i$$

$$\text{s.t.} \quad \min_{\Lambda} \frac{1}{k} \sum_{j=1}^{k} \max_{i \in \Lambda \cap K_j} \beta_i + \max_{i \in \bar{\Lambda} \cap K_j} \alpha_i \leq r \tag{1}$$

$$\text{and } 0 \leq \alpha_i, \beta_i, \leq 1, \quad i = 1, \ldots m. \tag{2}$$

Assume that the minimizing set Λ in condition (1) includes all the relay nodes (i.e. $\bar{\Lambda} = \emptyset$). Then the condition $\sum_{j=1}^{k} \max_{i \in K_j} \beta_i \leq kr$ implies that $\sum_{i=1}^{m} \beta_i \leq mr$, which in turn implies that $d_{QMF}(r) \geq m(1-r)$. Similarly, for all other Λ's by first relaxing the condition (1) to

$$\min_{\Lambda} \frac{1}{k} \sum_{j=1}^{k} \max\big(\max_{i \in \Lambda \cap K_j} \beta_i, \max_{i \in \bar{\Lambda} \cap K_j} \alpha_i\big) \leq r,$$

one can show that the sum of the m of the $2m$ variables α_i and β_i is smaller than mr. This again implies that $d_{QMF}(r) \geq m(1-r)$ which is the full-duplex performance. Therefore, QMF with the particular chosen fixed schedule achieves the optimal DMT. The result can be extended to the case with a direct link between the source and the destination.

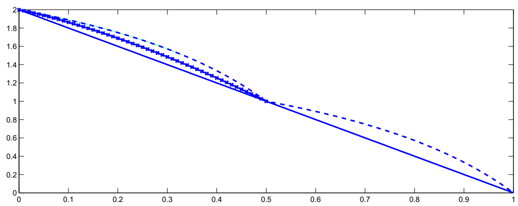

Fig. 5. '–' QMF with half-Rx half-Tx schedule for the relay, '- -' upper bound, '+' dynamic QMF.

V. DYNAMIC QMF

In the dynamic QMF (DQMF) strategy relays listen for a fraction of time, which is a function of their received CSI and then quantize, map, and forward the received signal as in the original QMF [1]. Clearly this generalizes DDF as well, since if the receive time is sufficient to decode, the difference between decoding and quantizing the received signal is the removal or forwarding of the additive noise correspondingly, which does not matter at high-SNR. It is applicable to general networks, but all the difficulty is then encapsulated in the (dynamic) choice of the receive times. To understand whether this additional flexibility is critical, we focus on a specific network.[2]

We consider the setup in Figure 2-(b) where S communicates to D with the help of two parallel half-duplex relays. For this network, we demonstrate that the DQMF strategy outperforms both DDF and the static QMF for $0 \leq r \leq 1/2$. See Figure 5.

1) QMF with fixed RX-TX schedules: The performance of the QMF strategy when the the relays listen for a fraction t_1 and t_2 of the time and transmit a fraction $(1-t_1)$ and $(1-t_2)$ respectively is lower bounded in [1, Section VIII-C] by $R_{QMF} \geq C_{h.d.}(t_1,t_2) - \kappa$ where κ is a constant independent of SNR and $C_{h.d.}(t_1,t_2)$ is given by

$$\begin{aligned} &C_{h.d.}(t_1,t_2) = \\ &\min\left(t_1 \log(1+|h_{s1}|^2\text{SNR}), (1-t_1)\log(1+|h_{1d}|^2\text{SNR})\right) \\ &+ \min\left(t_2 \log(1+|h_{s2}|^2\text{SNR}), (1-t_2)\log(1+|h_{2d}|^2\text{SNR})\right), \end{aligned}$$

where h_{si} and h_{id} are the channel coefficients between relay $i = 1,2$ and the source and the destination node respectively. In the high-SNR limit, the outage event is given by

$$\mathcal{O}(r) = \left\{ \min\left(t_1\alpha, (1-t_1)\gamma\right) + \min\left(t_2\eta, (1-t_2)\beta\right) \leq r \right\}$$

in terms of the multiplexing gains of the channels $\alpha = \lim_{\text{SNR}\to\infty} \frac{\log(1+|h_{s1}|^2\text{SNR})}{\log \text{SNR}}$ indicated in Figure 2-(b). Due to the symmetry of the two stages of communication it is easy to observe that the optimal fixed schedule for each relay (which does not depend on the instantaneous multiplexing gains $\alpha, \gamma, \beta, \eta$ of the channels) is to listen half the time and transmit the second half. Therefore, the DMT achieved by this strategy is given by the following optimization problem

$$\begin{aligned} d_{QMF}(r) = \min \quad & 4 - \alpha - \gamma - \eta - \beta \\ \text{such that} \quad & \min\left(\frac{1}{2}\alpha, \frac{1}{2}\gamma\right) + \min\left(\frac{1}{2}\eta, \frac{1}{2}\beta\right) < r. \end{aligned}$$

Solving this optimization problem we obtain

$$d_{QMF}(r) = 2 - 2r.$$

The dominant outage event occurs when one of two channels in each path are strong, say $\alpha = \eta = 1$, and the remaining two channels have a total of $2r$ multiplexing gain, i.e $\gamma + \beta = 2r$.

2) Upper Bound on the DMT Trade-off: In [14, Section VI], we derive an upper bound on the capacity of half-duplex relay networks, when all the channels are globally known, which allows to optimize the transmit and receive schedule for the relays based on global knowledge of instantaneous channel realizations. For the setup in Figure 2-(b), this upper bound yields

$$C \leq \max_{\substack{t_1(h_{s1},h_{s2},h_{2d},h_{2d}) \\ t_2(h_{s1},h_{s2},h_{2d},h_{2d})}} C_{h.d.}(t_1,t_2) + G$$

where G is a constant independent of SNR and t_1 and t_2 can be optimized as a function of the channel gains. In terms of the multiplexing gain r_C of the system, we have

$$r_C \leq \max_{\substack{t_1(\alpha,\gamma,\eta,\beta) \\ t_2(\alpha,\gamma,\eta,\beta)}} \min(t_1\alpha, (1-t_1)\gamma) + \min(t_2\eta, (1-t_2)\beta).$$

The right hand side is maximized by the choice $t_1 = \gamma/(\alpha+\gamma)$ and $t_2 = \beta/(\eta+\beta)$. Therefore, we get the following upper bound on the DMT of parallel channel in Fig. 1-(b),

$$\begin{aligned} d_{u.b.}(r) = \min \quad & 4 - \alpha - \gamma - \eta - \beta \\ \text{such that} \quad & \frac{\alpha\gamma}{\alpha+\gamma} + \frac{\eta\beta}{\eta+\beta} \leq r. \end{aligned}$$

Note that for $0 \leq r \leq 1/2$, $\alpha = 1$, $\eta = 1$, $\beta = r/(1-r)$ and $\gamma = 0$ (or any symmetrical configuration), and for $1/2 \leq r \leq 1$, $\alpha = \eta = \beta = 1$ and $\gamma = (r-1/2)/(3/2-r)$ are in the domain of the optimization problem above. Therefore, plugging these points provides immediately an upper bound on $d_{u.b.}(r)$, which in turn upper bounds $d(r)$, the DMT of the network. We have,

$$d(r) \leq d_{u.b} \leq \begin{cases} 2 - \frac{r}{1-r} & 0 \leq r < \frac{1}{2} \\ 1 - \frac{r-1/2}{3/2-r} & \frac{1}{2} \leq r \leq 1. \end{cases}$$

This upper bound is depicted in Figure 5.

3) Dynamic Decode and Forward: It is easy to verify that in this case, DDF has the same performance as the static QMF. In DDF, each relay waits until it is able to decode the transmitted message, *i.e.,* for the first relay $t_1 = r/\alpha$ and for the second relay $t_2 = r/\eta$. The scheme is in outage as soon as both $\alpha < r$ and $\eta < r$, in which case none of the relays ever get to transmit. Therefore, for the DMT of DDF, we have $d_{DDF}(r) \leq 2 - 2r$. An alternative strategy could be to split the

[2] In the reference [13] pointed to us in the review process a dynamic QMF strategy based on [1, Section VIII-C] is proposed where the switching times are optimized based on the entire network state as in [1, Section VIII-C]. In our terminology, we only use received CSI at the relays for determining the switching schedules and hence what we term as DQMF is different from [13].

information stream into two streams each of multiplexing gain $r/2$ and send them over the two orthogonal paths while relays perform DDF of the individual streams, i,e $t_1 = r/2\alpha$ and $t_2 = r/2\eta$. Communication is in outage if one of the streams is in outage which happens when $\alpha < r/2$. This means the diversity at rate r is $1 - r/2$, even worse.

4) Dynamic QMF: Now we want to show that a dynamic QMF strategy, where the RX times for the two relays depend on the backward channel realizations can perform better than the fixed half-half schedule when $0 \leq r \leq 1/2$. Consider the following dynamic QMF strategy. For a fixed x, let the first relay listen for a fraction $t_1 = \frac{x}{x+\alpha}$ of the total duration for communication, then quantize its received signal, map it to a new codeword and transmit it in the remaining $(1 - t_1)$ fraction of the time. Similarly, the second relay listens for a fraction $t_2 = \frac{x}{x+\eta}$ of the total time. Note that because t_1 and t_2 depend on the backward channel realizations α and η, this is a dynamic strategy. When contrasted with the upper bound in part-2 above, the relays here decide on their Rx times by assuming that the channels at the second stage have multiplexing gain x. (We assume that this forward channel information is not available at the relays, otherwise they could simply choose the x's as γ and β respectively as in the upper bound derivation and achieve the upper bound derived in part-2.) x will be later optimized as a function of r to achieve the best performance at every r. For now, it is a fixed constant such that $2r \leq x \leq 1$.

The motivation behind this strategy is the following: when the channels in the first stage are weak (in particular $2r$ or smaller as in the outage event for the fixed half-half schedule), this strategy allocates more time to the first stage. On the other hand, if they are strong (larger than x), the strategy allocates more time for the second stage which helps in case the second stage turns out to be weak. Therefore, it tries to balance the two stages of the communication by looking only at the realization of the backward channel at the relays. The RX times do not necessarily allow the relays to be able to decode the source message, therefore the strategy does not reduce to DDF.

The performance of the QMF strategy established in [1, Section VIII-C] also holds under dynamic schedules. DMT of this strategy is given by

$$d_{DQMF}(r) = \min \quad 4 - \alpha - \gamma - \eta - \beta \quad \text{s. t.} \tag{3}$$
$$\min\left(\frac{x}{x+\alpha}\alpha, \frac{\alpha}{x+\alpha}\gamma\right) + \min\left(\frac{x}{x+\eta}\eta, \frac{\eta}{x+\eta}\beta\right) \leq r.$$

The following proposition lower bounds $d_{DQMF}(r)$.

Proposition 5.1:

$$d_{DQMF}(r) \geq \min(2 - xr - r, 2 - \frac{xr}{x-r})$$

From the proposition, the DMT for DQMF is maximized when x is chosen such that $x^2 - rx - r = 0$. The resulting performance is depicted in Fig. 5.

We skip the proof of the proposition due to space constraints and below only summarize the dominant outage events considering the following three cases separately: the information transfer is limited by 1) the first hop for both relays; 2) the second hop for both relays; 3) the first hop for the first relay and the second hop for the second relay. These cases correspond to the following conditions

1) $\gamma \geq x$ and $\beta \geq x$: In this case the condition (3) becomes $(x/(x+\alpha))\alpha + (x/(x+\eta))\eta \leq r$. The dominant outage event is $\alpha = 0$ and $\eta = rx/(x-r)$ (or vice a versa), $\gamma = \beta = 1$, which yields $d_{DQMF}(r) = 2 - rx/(x-r)$.
2) $\gamma \leq x$ and $\beta \leq x$: In this case the condition (3) becomes $(\alpha/(x+\alpha))\gamma + (\eta/(x+\eta))\beta \leq r$. The dominant outage event is $\alpha = 1$ and $\eta = 1$, $\beta = 0$ and $\gamma = r(x+1)$, which yields $d_{DQMF}(r) = 2 - r(x+1)$.
3) $\gamma \geq x$ and $\beta \leq x$: In this case the condition (3) becomes $(x/(x+\alpha))\alpha + (\eta/(x+\eta))\beta \leq r$. The dominant outage events are $\alpha = 0$ and $\eta = 1$, $\beta = r(x+1)$ and $\gamma = 1$, and $\alpha = rx/(x-r)$ and $\eta = 1$, $\gamma = 1$, and $\beta = 0$, which yield $d_{DQMF}(r) = \min(2 - rx/(x-r), 2 - r(x+1))$. Note that the case $\gamma \leq x$ and $\beta \geq x$ is analogous due to the symmetry in the problem.

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