On Degrees-of-Freedom of Full-Duplex Uplink/Downlink Channel

Achaleshwar Sahai, Suhas Diggavi and Ashutosh Sabharwal

***Abstract*—Feasibility of full-duplex opens up the possibility of applying it to cellular networks to operate uplink and downlink simultaneously for multiple users. However, simultaneous operation of uplink and downlink poses a new challenge of intra-cell inter-node interference. In this paper, we identify scenarios where inter-node interference can be managed to provide significant gain in degrees of freedom over the conventional half-duplex cellular design.**

I. Introduction

Full-duplex is the ability to transmit and receive simultaneously in the same channel. Recently, significant progress has been made in showing the feasibility of wireless full-duplex [1–5], with experimental evidence displaying that two full-duplex nodes engaged in bi-directional communication enjoy significant rate gains over half-duplex. Rate gains from full-duplex in bi-directional communication are evident as full-duplex can have as much as twice (of half-duplex) the spatial degrees of freedom (DoF) to communicate. The feasibility of multi-antenna full-duplex [3–5] opens up the possibility of applying full-duplex to cellular networks to operate multiple uplink and downlink transmissions together. However, it is not immediately obvious whether the ability to use full-duplex will provide DoF gains in multi-user cellular networks.

The new challenge introduced by full-duplexing uplink and downlink transmissions can be understood by considering the example of a network which has one base-station and three mobile-stations, as shown in Fig. 1, all of which can use full-duplex. Due to simultaneous uplink and downlink, the uplink transmissions from MS_2 and MS_3 interfere with the downlink reception at MS_1. Similar inter-node interference also affects the downlink reception at MS_2 and MS_3. Note that, while communicating in half-duplex, the uplink and downlink transmissions happen on separate channels, thus circumventing any inter-node interference. The new intra-cell inter-node interference created by full-duplexing uplink and downlink transmissions together needs to be managed in order to leverage full-duplex gains in multi-user cellular networks.

In this paper, we study a network composed of a single $L-$antenna full-duplex base-station which communicates in downlink and uplink with K single-antenna full-duplex mobile stations. We call such a network an (L, K) full-duplex network. For an (L, K) full-duplex network, we show that significant gain in DoF is possible when the transceivers are capable of operating in full-duplex mode instead of half-duplex mode. We show gain in DoF under the assumption that channels undergo ergodic phase fading [6] and instantaneous channel state information (CSI) of all the channels is available to all the nodes in the network. Specifically, we show that whenever $K > L$, then the DoF in uplink and downlink of the full-duplex network is more than the half-duplex network. Further, we show that if $K \geq 2L$, then full-duplex operation doubles the total DoF of the network compared to half-duplex.

Recognizing that both ergodic phase fading channels and instant CSIT are rarely available in practice, we study a (2,2) full-duplex network where channels are fast-fading and all the nodes have delayed CSIT [7]. Interestingly, for such a network too, we find that operating in full-duplex offers significant gain in DoF compared to half-duplex, thus making full-duplex an attractive alternative to half-duplex in some multi-user cellular networks.

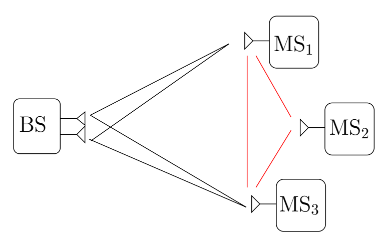

Fig. 1. Network with 2 antenna base-station and 3 single antenna mobile stations. The inter-node interference is between the mobile stations is also highlighted.

The achievability scheme when instantaneous CSIT is available to all nodes and the channels are ergodic phase fading is based on the ergodic alignment scheme proposed in [6]. The key idea is to bin the transmissions over *complimentary* channel realizations such that the inter-node interference occurring at the mobile receivers during one time slot can be completely negated by the inter-node interference occurring at another time slot. By combining transmissions over suitably chosen time slots, we can effectively create a network free of any inter-node interference, thus decoupling the effect of uplink on downlink.

For the two mobile user delayed CSIT case, we present a modified version of the retrospective alignment scheme proposed in [7, 8] to accommodate uplink transmissions along with downlink transmissions. The retrospective interference alignment scheme in [7, 8] treats prior downlink transmissions as side-information for future decoding, which we modify

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such that the downlink transmission combined with inter-node interference is treated as side-information for future decoding.

In order to find a tight outer-bound, we recognize that in a single base-station cellular network, due to inter-node interference the total DoF of uplink transmission from one mobile user and downlink reception at another mobile user is limited to 1.

Related Work: In [9], the authors consider a fully connected full-duplex network, with instantaneous CSIT, in which every source has a message for every destination, for which they show that the sum-DoF can be at most twice of the half-duplex counterpart. We differ from [9] in two aspects. First, unlike [9], the network we consider does not have messages from every source to every destination, thus their achievability is not directly applicable. Secondly, we provide a tight characterization of the DoF in uplink and downlink, while [9] concentrates on sum-DoF. The authors in [10] also consider full-duplex interaction between nodes, but unlike this paper, they consider a network where source and sinks are disjoint.

The rest of the paper is organized as follows. Section II describes the system model. Section III presents the main results of the paper. Section IV is dedicated to strategies and outer-bound for ergodic phase fading channels with instantaneous CSIT and Section V describes the achievability and outer-bound of fast-fading channels with delayed CSIT. Section VI concludes the paper.

II. System Model

In an (L, K) full-duplex network, full-duplex capability implies that the base-station (mobile-station) can transmit and receive from all the L (single) antennas simultaneously. The k^{th} mobile station intends to transmit an uplink message $W_{\text{UL},k}$ and receive a downlink message $W_{\text{DL},k}$ to/from the base-station at rates $R_{\text{UL},k}$ and $R_{\text{DL},k}$ respectively.

A. Channel Model

Uplink: Let $u_k[n] \in \mathbb{C}$, with power constraint $\mathbb{E}(|u_k[n]|^2) \leq P$, be the signal transmitted by the k^{th} mobile station and $\mathbf{u}[n] = \left[u_1[n], \ldots, u_K[n]\right]^T$. Then, the received signal at the base-station in the n^{th} time slot denoted by $\mathbf{y}_{\text{BS}}[n] \in \mathbb{C}^L$ is given by

$$\mathbf{y}_{\text{BS}}[n] = \sum_{k=1}^{K} \mathbf{H}_{\text{up},k}[n] u_k[n] + \mathbf{z}_{\text{BS}}[n], \tag{1}$$

where $\mathbf{H}_{\text{up},k}[n] \in \mathbb{C}^L$ is a column vector denoting the uplink channel matrix from the k^{th} mobile station to the base-station, and $\mathbf{z}_{\text{BS}}[n] \in \mathbb{C}^L$ is Gaussian noise each of whose entries are drawn from the $\mathcal{CN}(0, 1)$ distribution.

Downlink: Let the signal transmitted from the base-station be denoted by $\mathbf{d}[n] \in \mathbb{C}^L$ which is power constrained such that $\mathbb{E}(|\mathbf{d}[n]|^2) \leq KP$. Then, the received signal at the k^{th} mobile station, $y_{\text{MS},k}$, which is a combination of the signal from the base-station and interference from the rest of the mobile stations is given by

$$y_{\text{MS,k}}[n] = (\mathbf{H}_{\text{down},k}[n])^T \mathbf{d}[n] + (\mathbf{H}_{\text{I},k}[n])^T \mathbf{u}[n] + z_k[n], \tag{2}$$

where $\mathbf{H}_{\text{down},k}[n]$ is column vector denoting the downlink channel matrix, and $\mathbf{H}_{\text{I},k}[n] \in \mathbb{C}^K$ is the column vector representing the inter-node interference channel to the k^{th} mobile station. Since self-interference is assumed to be completely suppressed, the k^{th} entry of $\mathbf{H}_{\text{I},k}[n]$ is fixed to 0. Let

$$\begin{aligned} \mathbf{H}_{\text{up}}[n] &= \left[\mathbf{H}_{\text{up},1}[n] \ldots \mathbf{H}_{\text{up},K}[n]\right] \\ \mathbf{H}_{\text{down}}[n] &= \left[\mathbf{H}_{\text{down},1}[n] \ldots \mathbf{H}_{\text{down},K}[n]\right]. \\ \mathbf{H}_{\text{I}}[n] &= \left[\mathbf{H}_{\text{I},1}[n] \ldots \mathbf{H}_{\text{I},K}[n]\right]. \end{aligned} \tag{3}$$

For the fast-fading channel model each entry of $\mathbf{H}_{\text{up}}[n]$, $\mathbf{H}_{\text{down}}[n]$ and the non-diagonal entries of $\mathbf{H}_{\text{I}}[n]$ are independent from each other and are assumed to be i.i.d. over time with magnitude having variance 1. For ergodic phase fading, additionally, the phase of each channel coefficient is drawn from a uniform phase distribution indepedent of its magnitude (see equation (9) in [6] for details). Instantaneous CSIT implies that all nodes know $\mathbf{H}_{\text{up}}[n], \mathbf{H}_{\text{down}}[n]$ and $\mathbf{H}_{\text{I}}[n]$ prior to transmision at time n. Under delayed CSIT model, the channels at time n, i.e., $\mathbf{H}_{\text{up}}[n], \mathbf{H}_{\text{down}}[n]$ and $\mathbf{H}_{\text{I}}[n]$ are known to all nodes at time $n + 1$.

B. Encoding and Decoding

Let $\mathbf{y}_{\text{BS}}^{n-1} \triangleq (\mathbf{y}_{\text{BS}}[1], \ldots, \mathbf{y}_{\text{BS}}[n-1])$. Then, at the base-station, encoding in the n^{th} time slot is

$$\mathbf{d}[n] = f_{n,\text{BS}}(W_{\text{DL},1}, \ldots, W_{\text{DL},K}, \mathbf{y}_{\text{BS}}^{n-1}).$$

Similarly, at the k^{th} mobile station, the encoding is

$$u_k[n] = f_{n,\text{MS},k}(W_{\text{UL},k}, y_{\text{MS},k}^{n-1}).$$

Note that the encoding also depends on the type of CSIT available, although the dependence is not explicitly shown here. The decoding function at a terminal recovers the message/s it intends to receive after N transmissions, i.e., at the base-station and the k^{th} mobile station, we have

$$(\widehat{W}_{\text{UL},1}, \ldots, \widehat{W}_{\text{UL},K}) = g_{\text{BS}}(\mathbf{y}_{\text{BS}}^N), \tag{4}$$

$$\widehat{W}_{\text{DL},k} = g_{\text{MS},k}(y_{\text{MS},k}^N). \tag{5}$$

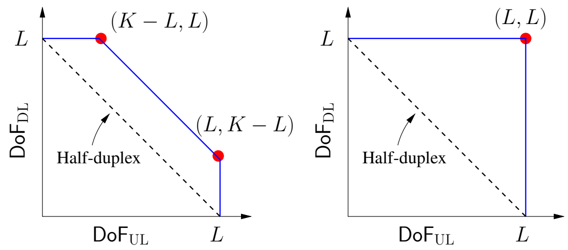
The probability of error is defined as the probability that at least one of the messages is not decoded correctly,

$$p_{\text{error}} = p\left(\bigcup_{X \in \{\text{UL},\text{DL}\},k} \widehat{W}_{X,k} \neq W_{X,k}\right) \tag{6}$$

The rate tuple in uplink $(R_{\text{UL},1}, \ldots, R_{\text{UL},K})$ and in downlink $(R_{\text{DL},1}, \ldots, R_{\text{DL},K})$ are said to be achievable if there exist encoding and decoding functions such that $p_{\text{error}} \to 0$ as $N \to \infty$. The DoF in uplink and downlink are defined as

$$\mathsf{DoF}_{\text{UL}} = \lim_{P \to \infty} \frac{\sum_{i=1}^{K} R_{\mathsf{UL},k}}{\log P}. \tag{7}$$

The DoF in downlink, DoF_{DL} is defined similarly. The sum-total of DoF of the network is $\mathsf{DoF}_{\text{UL}} + \mathsf{DoF}_{\text{DL}}$.



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(a) $L < K < 2L$ (b) $K \geq 2L$

Fig. 2. The DoF region in full-duplex network with L antenna base-station and K single antenna mobile-station

III. Results

Theorem 1 (Instantaneous CSIT and ergodic fading): The set of all achievable pairs $(\mathsf{DoF}_{\mathrm{UL}}, \mathsf{DoF}_{\mathrm{DL}})$ is given by

$$\mathsf{DoF}_{\mathrm{UL}} \leq \min(L, K) \quad (8a)$$

$$\mathsf{DoF}_{\mathrm{DL}} \leq \min(L, K) \quad (8b)$$

$$\mathsf{DoF}_{\mathrm{UL}} + \mathsf{DoF}_{\mathrm{DL}} \leq \min(2L, K) \quad (8c)$$

Remark 1: Theorem 1 shows that for $K > L$, full-duplex has larger DoF-region than half-duplex, as shown in Fig. 2. For $K \geq 2L$, the maximum achievable square region of $(\mathsf{DoF}_{\mathrm{UL}}, \mathsf{DoF}_{\mathrm{DL}})$ is realized which is twice as big as the region with half-duplex time-sharing. Therefore, full-duplex capability at all nodes allows the base-station to accomodate twice as many mobile users as half-duplex in the DoF sense.

Theorem 2 (Delayed CSIT fast-fading): The DoF pair achievable with delayed CSIT in a $(2, 2)$ full-duplex network is given by

$$\mathsf{DoF}_{\mathrm{DL}} \leq 4/3 \quad (9a)$$

$$\mathsf{DoF}_{\mathrm{UL}} \leq 2 \quad (9b)$$

$$\mathsf{DoF}_{\mathrm{DL}} + \mathsf{DoF}_{\mathrm{UL}} \leq 2 \quad (9c)$$

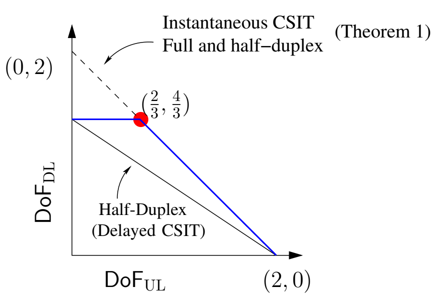

Fig. 3. DoF region for $(2, 2)$ full-duplex network with delayed CSIT.

Remark 2: Full-duplex (with delayed CSIT) has a larger rate-region than half-duplex (with delayed CSIT) with independent operation in uplink and downlink as shown in Fig. 3. From Fig. 3, we also observe that for the $(2, 2)$ network, delayed CSIT full duplex region is smaller than instantaneous CSIT half-duplex region, which is evident from the fact that with instantaneous CSIT half-duplex and full-duplex regions are identical.

IV. Instantaneous CSIT with Ergodic Fading

A. Achievability

We describe the key ideas of achievability through an example of full-duplex network with $K = 2L$, and provide pointers to the achievability for the arbitrary case.

When $K = 2L$, the DoF-region is a square as shown in Fig. 2(b) whose non-trivial corner point is (L, L). The key to achieve the non trivial corner point is to deliver $K = 2L$ independent messages, both uplink as well as downlink, in exactly two time slots. We start with generation of K independent symbols $x_1, x_2 \dots x_K$ as downlink symbols, where x_k is intended for the k^{th} user. Similarly, each of the mobile stations generates an independent symbol u_k intended for uplink transmission. Let $\mathbf{x} = [x_1, \dots, x_K]^T$ and $\mathbf{u} = [u_1, \dots, u_K]^T$.

Due to limited spatial resources at the base-station, $K = 2L$ independent messages cannot be delivered, either in uplink or downlink in one time slot. Instead, we pair two time slots with complimentary channel realizations such that the inter-node interference during one time slot is negated by the other time slot. Let the time slot t_1 be paired with time slot $t_2 > t_1$, then we specify the constraints that the channel matrices at time t_1 and t_2 must satisfy.

Negating the interference: It is possible to negate the inter-node interference generated in time slot t_1 by the inter-node interference generated in time slot t_2, if the channel realizations at time t_1 and t_2 satisfy the following constraint:

$$\mathbf{H}_{\mathrm{I}}[t_2] = \mathbf{H}_{\mathrm{I}}[t_1]. \quad (10)$$

To ensure that the interference generated in time slot t_1 can be negated by the interference generated in time slot t_2, we have to set the $\mathbf{u}[t_1] = \mathbf{u}[t_2] = \mathbf{u}$. Then, at the k^{th} mobile user, by subtracting the received signal in time slot t_2 from the received signal in time slot t_2, we get

$$y_{\mathrm{MS},k}[t_1] - y_{\mathrm{MS},k}[t_2] = (\mathbf{H}_{\mathrm{down},k}[t_1])^T \mathbf{d}[t_1] - (\mathbf{H}_{\mathrm{down},k}[t_2])^T \mathbf{d}[t_2] + z_k[t_1] - z_k[t_2], \quad (11)$$

which is free of inter-node interference.

Zero-Forcing for downlink: Note that, so far we have not specified how to design $\mathbf{d}[t_1]$ and $\mathbf{d}[t_2]$. Ignoring the noise, we can rewrite (11) as

$$y_{\mathrm{MS},k}[t_1] - y_{\mathrm{MS},k}[t_2] = \left[(\mathbf{H}_{\mathrm{down},k}[t_1])^T \; (-\mathbf{H}_{\mathrm{down},k}[t_2])^T \right] \begin{bmatrix} \mathbf{d}[t_1] \\ \mathbf{d}[t_2] \end{bmatrix}. \quad (12)$$

Somehow, if the vectors $\mathbf{d}[t_1]$ and $\mathbf{d}[t_2]$ are designed such that

$$\begin{bmatrix} \mathbf{d}[t_1] \\ \mathbf{d}[t_2] \end{bmatrix} = \left[(\mathbf{H}_{\mathrm{down}}[t_1])^T \; (-\mathbf{H}_{\mathrm{down}}[t_2])^T \right]^{-1} \mathbf{x}. \quad (13)$$

Then, each mobile user can jointly process the received symbols in time slot t_1 and t_2, like (12), to obtain a noisy version of the symbol intended for itself. However, the design of downlink vectors, as in (13), is possible only if at time t_1, the base-station knows both $\mathbf{H}_{\mathrm{down}}[t_1]$ and $\mathbf{H}_{\mathrm{down}}[t_2]$. To avail base-station of such knowledge, we can further constrain to pick time slots t_1 and t_2 such that channel matrix $\mathbf{H}_{\text{down}}[t_2]$ is deterministically related to $\mathbf{H}_{\text{down}}[t_1]$. Moreover, the inverse operation in (13) is feasible only if $\left[(\mathbf{H}_{\text{down}}[t_1])^T \ (-\mathbf{H}_{\text{down}}[t_2])^T\right]$ is full-rank. One (not necessarily unique) deterministic mapping from $\mathbf{H}_{\text{down}}[t_1]$ to $\mathbf{H}_{\text{down}}[t_2]$ which also preserves the full-rank condition is the following

$$\mathbf{H}_{\text{down},k}[t_1] = \omega^{k-1}\mathbf{H}_{\text{down},k}[t_2]. \tag{14}$$

where ω is K^{th} root of unity. Thus, in addition to (10), we impose constraint (14) on channel matrices to pair time slots t_1 and t_2. Under constraints (10) and (14), by performing (12), each mobile station can recover a noisy version of the message intended for itself in two time slots, i.e., overall downlink $\mathsf{DoF}_{\text{DL}} = K/2 = L$.

Sufficient equations for uplink: Finally, the base-station can decode $K = 2L$ independent uplink messages using the received symbols in two time slots, if $\mathbf{y}_{\text{BS}}[t_1]$ and $\mathbf{y}_{\text{BS}}[t_2]$ represent $K = 2L$ linearly independent combinations of $u_1, u_2, \ldots, u_K$. To ensure linear independence, we can further constrain to pair t_1 and t_2 only when the uplink matrices are related as

$$\mathbf{H}_{\text{up},k}[t_1] = \omega^{k-1}\mathbf{H}_{\text{up},k}[t_2], . \tag{15}$$

With $K = 2L$ linearly independent equations in 2 time slots, all the $u_1, \ldots, u_K$ messages are decoded which implies uplink $\mathsf{DoF}_{\text{UL}} = K/2 = L$.

Now, we note that for ergodic phase fading channels, it is not possible to exactly satisfy (10), (14) and (15). However, (10), (14) and (15) can be *approximately* satisfied, as is the case with ergodic interference alignment with high probability such that for every t_1 there exists some $t_2 \in \mathbb{N}$ w.h.p (the existence of t_2 relies on the ergodic phase fading channel).

Remark 3: The two time slot strategy of ergodic alignment and downlink and uplink zero-forcing can be viewed as translating the (L, K) full-duplex network into a $(2L, K)$ interference-free half-duplex network. For any $K \geq L$, the DoF-region of $(2L, K)$ half-duplex network is larger than (L, K) half-duplex network and the factor of gain is twice for $K \geq 2L$, which is essentially the gain of full-duplex over half-duplex.

To achieve the non-trivial corner points when $L < K < 2L$, we have to slightly modify the strategy employed for $K = 2L$. The strategy will need pairing of K time slots $t_1, \ldots, t_K$ such that the interference channel matrix in each of time slots is approximately the same (to negate inter-node interference). Moreover, the uplink/downlink matrices must be deterministic mapping of the uplink/downlink matrix at time slot t_1, such that downlink transmission vectors can be designed for downlink zero-forcing and sufficient linearly independent equations are available for uplink decoding. Due to lack of space, we omit the details of uplink/downlink transmissions.

B. Outer Bound

The bounds (8a) and (8b) are trivial bounds obtained by letting all the mobile users cooperate among themselves to form a MIMO two-way network. The $(\mathsf{DoF}_{\text{UL}}, \mathsf{DoF}_{\text{DL}})$ pair of the (L, K) full-duplex network is upper bounded by the DoF of MIMO in each direction which is the minimum of transmit and receive antennas, $\min(L, K)$ [11].

To prove (8c), we adopt the following strategy. Except two messages W_{UL,k_1} and W_{DL,k_2} for $k_1 \neq k_2$, we set all the other messages to null. Then, we show that for the messages that are not set to null

$$\mathsf{DoF}_{\text{UL},k_1} + \mathsf{DoF}_{\text{DL},k_2} \leq 1. \tag{16}$$

Adding all bounds of the type (16) for all k_1, k_2, we get

$$\begin{aligned} K(\mathsf{DoF}_{\text{UL}} + \mathsf{DoF}_{\text{DL}}) &\leq K^2 \\ \implies \mathsf{DoF}_{\text{UL}} + \mathsf{DoF}_{\text{DL}} &\leq K. \end{aligned} \tag{17}$$

The bound (8c) is immediately obtained by combining (17), (8a) and (8b). Thus to prove (17), without loss of generality, we choose $k_1 = 1$ and $k_2 = K$ and prove (16) as follows.

Transformation to equivalent half-duplex network: Inspired by the proof of outer-bound in [9], we transform the (L, K) full-duplex network into an equivalent $K+1 \times K+1$ half-duplex network as shown in Fig. 4. We construct the half-duplex network in such a way that corresponding to each node N in the (L, K) full-duplex network, there is a transmitter N_{T} and a receiver N_{R} in the $(K+1) \times (K+1)$ half-duplex network. The half-duplex nodes N_{T} and N_{R} have the same number of antennas as N. Further, N_{T} and N_{R} are provided all the messages that the node N wishes to transmit.

The input-output relationship in the half-duplex network is such that the channel matrix in the time slot n from transmitter N_{T} to the receiver M_{R} is identical to the channel matrix from node N to M in the (L, K) full-duplex network. The noise variances at the receiver N_{R} in the half-duplex network are identical to the noise variances at node N in the full-duplex network. Finally, the encoding strategy in the half-duplex network is such that in the n^{th} time slot, the signal transmitted by node N_{T} is a function of the message/s to be transmitted by N_{T} and the past channel output observations at node N_{R}. The decoding function at node N_{R} maps all the channel output observations at N_{R} to the message/s intended for N_{R}. With the properties described above, the $(K+1) \times (K+1)$ half-duplex network is equivalent to (L, K) full-duplex network in the sense that any rate-tuple achievable in the full-duplex network is also achievable in the half-duplex network.

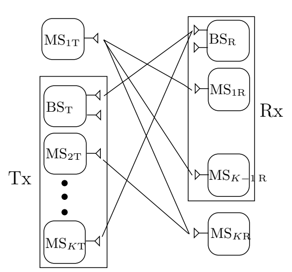

Fig. 4. The 4 node X transformation of an (L, K) full-duplex network

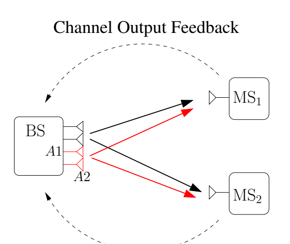

Fig. 5. $(2, 2)$ full-duplex network w/o uplink messages is enhanced to a $(4, 2)$ half-duplex network with channel output feedback

Transformation to 4 node X network: The $K+1 \times K+1$ half-duplex network can be further transformed into a 4 node X network by creating a virtual transmitter Tx by allowing $\mathrm{BS_T}, \mathrm{MS_{2T}}, \ldots, \mathrm{MS}_{K\mathrm{T}}$ fully cooperate among themselves and virtual receiver Rx by letting $\mathrm{BS_R}, \mathrm{MS_{1R}}, \ldots, \mathrm{MS}_{K-1\ \mathrm{R}}$ fully cooperate among themselves as shown in Fig. 4. Since cooperation can only enhance the capacity region, thus the achievable DoF of the 4 node X network in Fig. 4 is an upper bound of the $(K+1) \times (K+1)$ half-duplex network.

Nulling all but two messages: Now, in our 4 node X network, we set all the messages to null, except $W_{\mathrm{UL},1}$ and $W_{\mathrm{DL},K}$. The upper bound on the sum $\mathsf{DoF}_{\mathrm{UL},1} + \mathsf{DoF}_{\mathrm{DL},K}$ in the absence of any other message in the network is definitely an upper bound on the sum in the presence of messages other than $W_{\mathrm{UL},1}$ and $W_{\mathrm{DL},K}$. We can now borrow the outer-bound for the 4 node X network shown in [9] (see Fig. 5 of on Page 9 of [9]) as an outer bound for our 4 node X network shown in Fig. 4, which is

$$\mathsf{DoF}_{\mathrm{UL},1} + \mathsf{DoF}_{\mathrm{DL},K} \leq 1. \quad (18)$$

At every step of the transformation of the original (L, K) full-duplex network, we have only enhanced the network. Thus, (18) applies to the original (L, K) full-duplex network as well, which is what we intended to prove.

V. Delayed CSIT with Fast Fading

A. Achievability

With delayed CSIT and no uplink messages, a $(2, 2)$ network can achieve $\mathsf{DoF}_{\mathrm{DL}} = \frac{4}{3}$ as shown in [7]. Also, with delayed (or no) CSIT and no downlink messages, the $(2, 2)$ network can trivially achieve $\mathsf{DoF}_{\mathrm{UL}} = 2$. Therefore, the DoF pair $(2/3, 4/3)$ is the only non-trivial corner point whose achievablity needs to be shown. We use a 3 time slot strategy to deliver 4 downlink and 2 uplink messages. In the first two time slots, the downlink vectors that are transmitted are

$$\mathbf{d}[1] = \begin{bmatrix} x_{1,1} \\ x_{1,2} \end{bmatrix}, \ \mathbf{d}[2] = \begin{bmatrix} x_{2,1} \\ x_{2,2} \end{bmatrix}, \quad (19)$$

where $x_{k,i}$ is the i^{th} independent symbol intended for the k^{th} mobile-station. Since the first time slot has downlink messages dedicated only for MS_1, even the uplink message is only transmitted by MS_1. Similarly, in the second time slot uplink messages are transmitted only by MS_2. Therefore,

$$\mathbf{u}[1] = \begin{bmatrix} u_1 \\ 0 \end{bmatrix}, \ \mathbf{u}[2] = \begin{bmatrix} 0 \\ u_2 \end{bmatrix}. \quad (20)$$

Before the third time-slot, via delayed CSIT, the base-station is aware of all the channels states till the second time slot. Moreover, it would have decoded u_1 and u_2. Thus, the base-station can reconstruct the sum $y_{\mathrm{MS},1}[2] + y_{\mathrm{MS},2}[1]$ (within bounded noise variance) and transmit it on one its antennas in the third time slot. From the received message in the third time slot, MS_1 can recover $y_{\mathrm{MS},2}[1]$ and subtract the contribution of u_1 from it, thus obtaining a linear combination of $x_{1,1}$ and $x_{1,2}$. Together with $y_{\mathrm{MS},1}[1]$, MS_1 is now equipped with two linear combinations of the downlink messages $x_{1,1}, x_{1,2}$ and thus can decode them both. Similarly, MS_2 can decode $x_{2,1}, x_{2,2}$. The resulting DoF in downlink is $\mathsf{DoF}_{\mathrm{DL}} = 4/3$ and in uplink is $\mathsf{DoF}_{\mathrm{UL}} = 2/3$. Note that, the downlink transmissions in the first and second time slot are identical to the achievable strategy is [7].

B. Outer Bound

To outer-bound $\mathsf{DoF}_{\mathrm{DL}}$ in $(2, 2)$ full-duplex network, first we null all the uplink messages. Without any uplink messages, the transmissions from the MS_1 and MS_2 will depend only upon the past received symbols as defined in (4). Now, we consider an enhancement of the network and prove an outer bound for it. We provide two more antennas $A1$ and $A2$ as shown in Fig. 5. At all times, we constrain the channel from $A1$ to MS_2 to be identical to the channel from MS_1 to MS_2 and similarly the channel from $A2$ to MS_1 is identical to the channel from MS_2 to MS_1. For the enhanced network, we provide channel output feedback, from both the mobile stations, $y^{n-1}_{\mathrm{MS},1}$ and $y^{n-1}_{\mathrm{MS},2}$, to the base-station. Finally, at all time n, we constrain the transmission from $A1$ to depend only on $y^{n-1}_{\mathrm{MS},1}$ and from $A2$ to depend only on $y^{n-1}_{\mathrm{MS},2}$. Note that in the enhanced network, switching off the full-duplex capability of all the nodes will still allow all the strategies that are possible in the $(2, 2)$ full-duplex network with uplink messages nulled out. Now, the delayed CSIT outer bound (Theorem 2 from [7] and Section IV in [7]) can be applied to the enhanced network to outer-bound the $\mathsf{DoF}_{\mathrm{DL}}$, which turns out to be

$$\mathsf{DoF}_{\mathrm{DL}} \leq 4/3. \quad (21)$$

The bound (9b) is trivial and bound (9c) is evident from Theorem 1.

VI. Conclusion

In this paper, we study the DoF gain of using full-duplex instead of half-duplex in a single base-station uplink/downlink network. We show that if channels are ergodic phase fading and instantaneous CSIT is available, then the DoF of the full-duplex network can be twice as large as that of half-duplex if the number of mobile stations is twice the number of base-station antennas. For the delayed CSIT fast fading channel model, we show that for a $(2, 2)$ network, although full-duplex does not improve the maximum achievable uplink/dowlink DoF, it has a larger DoF-region than half-duplex.

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