The Interference-Multiple-Access Channel

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***Abstract*—We introduce the interference-multiple-access channel, which is a discrete memoryless channel with two transmitters and two receivers, similar to the interference channel. One receiver is required to decode the information encoded at one transmitter, the other receiver is required to decode the messages from both transmitters. We provide an inner bound on the capacity region of this channel, as well as an outer bound for a special class of such channels. For this class, we also quantify the gap between inner and outer bound and show that the bounds match for a semi-deterministic channel, providing a complete characterization. For the Gaussian case, we show that the gap is at most 1 bit, yielding an approximate characterization.**

I. Introduction

In many communication systems, the signals sent by two different transmitters get “mixed” by the channel, and the channel output observed by a receiver can depend on both these signals. This phenomenon occurs for instance in wireless point-to-point communication, when two transmitter-receiver pairs share the same wireless communication resources. However, communication need not be point-to-point. We encounter a variety of situations where transmitters broadcast information to several receivers and each receiver might be interested in the information coming from different transmitters, for instance in wireless networks. In this paper, we introduce an information-theoretic model for one such scenario which we call the interference-multiple-access channel.

The general two-user channel, shown in Figure 1 is a channel connecting two transmitters to two receivers. Assume that Transmitter 1 encodes a message W_1 and Transmitter 2 a message W_2. When Receiver 1 is only required to decode W_1 and Receiver 2 only W_2, then the communication problem is called the interference channel, because the signal sent by Transmitter 1 interferes with the communication between Transmitter 2 and Receiver 2 and vice versa. This problem was introduced in [6], and as of yet, the general capacity region for this channel is unknown. The best known inner bound was provided by Han and Kobayashi in [5]. See [1] for a survey of solved special cases. In particular, the results in [4] and [3] are related to our work. In [4], the capacity region of a class of deterministic interference channels is given, and [3] provides a 1 bit-approximation of the capacity region for the Gaussian interference channel. More recently, Telatar and Tse [7] generalized the results of [4] and [3].

A different problem arises when the receivers are required to decode both messages W_1 and W_2. In this case, for fixed product input distributions, we can achieve the intersection of the achievable multiple-access regions for Receiver 1 and Receiver 2. The capacity region for the channel is the union over all product input distributions of these intersections.

We introduce a new problem, where we require Receiver 1 to decode both W_1 and W_2, but Receiver 2 is only required to decode the message W_2 encoded by Transmitter 2. To the best of our knowledge, this problem formulation is new, and we call it the *interference-multiple-access* (IMA) channel. Our main results are an achievable region for the general IMA channel, an outer bound for a certain class of so-called structured IMA channels, as well as an expression for the gap between the inner and outer bound for structured IMA channels. This class of structured channels is the IMA analog of the class of interference channels considered in [4] and [7].

In Section II, we define the general and structured IMA channels in detail. Sections III and IV state our main results and Sections V and VI contain the corresponding proofs. We conclude in Section VII.

II. Problem Statement

A. The General Interference-Multiple-Access Channel

Consider the discrete memoryless channel shown in Figure 1, connecting two transmitters to two receivers. The input and output alphabets are discrete sets $\mathcal{X}_1$, $\mathcal{X}_2$, $\mathcal{Y}_1$ and $\mathcal{Y}_2$. If input symbols (x_1, x_2) are transmitted, the outputs (Y_1, Y_2) are governed by the conditional probability distribution $p_{Y_1,Y_2|X_1,X_2}(\cdot|x_1, x_2)$. Messages W_1 and W_2 are encoded at Transmitter 1 and 2, respectively, and we require Receiver 1 to produce estimates of (W_1, W_2), while Receiver 2 is only required to estimate W_2. We refer to this problem as the general interference-multiple-access (IMA) channel.

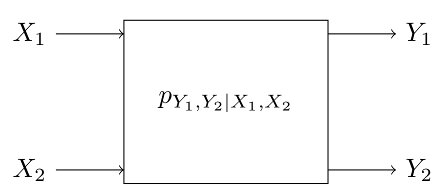

Fig. 1. The general two-user channel.

We define the capacity region in the usual way as follows.

Definition 1: The capacity region $\mathcal{R}$ of a given IMA channel is the set of all rate pairs (R_1, R_2) such that for any $\epsilon > 0$ there are encoders enc_1 and enc_2 of rates at least R_1 and R_2 respectively, and decoders dec_1 and dec_2 such that the error probability is at most ϵ.

Our first result, outlined in Section III, is the description of an inner bound on $\mathcal{R}$, *i.e.*, a region of pairs (R_1, R_2) that are guaranteed to be achievable.

B. The Structured Interference-Multiple-Access Channel

In Section IV, we consider a specific class of two-user channels, depicted in Figure 2, and described in the following definition.

Definition 2: A two-user channel is *structured* if the outputs Y_1 and Y_2 are determined from the input symbols $(x_1, x_2) \in \mathcal{X}_1 \times \mathcal{X}_2$ in the following way. First, x_j is fed through a discrete memoryless channel with transition probability $p_{S_j|X_j}$ to obtain a random variable S_j, taking values in the discrete alphabet $\mathcal{S}_j$, for $j = 1, 2$. Then, Y_1 and Y_2 are computed as the outputs of the functions $f_1(x_1, S_2)$ and $f_2(x_2, S_1)$, respectively. We assume that the functions f_j, $j = 1, 2$, have the property that for any fixed x_1, the map

$$f_1(x_1, \cdot) : \mathcal{S}_2 \to \mathcal{Y}_1, \ S_2 \mapsto Y_1$$

is invertible, and similarly for f_2. The channel is fully specified by $\mathcal{X}_j$, $\mathcal{S}_j$, $\mathcal{Y}_j$, $p_{S_j|X_j}$ and f_j, for $j = 1, 2$.

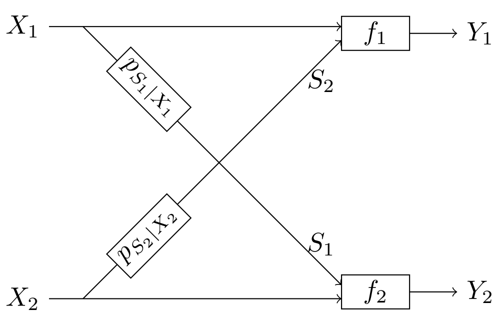

Fig. 2. The structured two-user channel.

The structured two-user channel was introduced in this general form by Telatar and Tse in [7], and its deterministic version was earlier studied by El Gamal and Costa [4]. Both [7] and [4] considered the interference version of this channel. Telatar and Tse provided a new outer bound on the rate-region of this problem, and quantified the gap between this outer bound and the Han-Kobayashi inner bound in [5].

In Section IV, we provide an outer bound on the capacity region $\mathcal{R}$ for the structured IMA channel. We quantify the gap between inner and outer bounds on the capacity region, using the techniques introduced in [7].

III. Inner Bound

Definition 3: For a given distribution $p_{Q,X_1,X_2,U_1} = p_Q \ p_{X_1|Q} \ p_{X_2|Q} \ p_{U_1|X_1,Q}$, define $\mathcal{R}_i(Q, X_1, X_2, U_1)$ as the set of all pairs (R_1, R_2) of non-negative real numbers satisfying the 6 constraints

$$\begin{aligned} R_1 &\leq I(X_1; Y_1|X_2, Q), && (1)\\ R_2 &\leq I(X_2; Y_1|X_1, Q), && (2)\\ R_1 + R_2 &\leq I(X_1, X_2; Y_1|Q), && (3)\\ R_2 &\leq I(X_2; Y_2|U_1, Q), && (4)\\ R_1 + R_2 &\leq I(U_1, X_2; Y_2|Q) + I(X_1; Y_1|U_1, X_2, Q), && (5)\\ R_1 + 2R_2 &\leq I(U_1, X_2; Y_2|Q) + I(X_1, X_2; Y_1|U_1, Q). && (6) \end{aligned}$$

In addition, define

$$\mathcal{R}_i \triangleq \cup_{Q,X_1,X_2,U_1} \mathcal{R}_i(Q, X_1, X_2, U_1),$$

where the union is over all distributions of the form $p_Q \ p_{X_1|Q} \ p_{X_2|Q} \ p_{U_1|X_1,Q}$. To simplify the presentation, we use the random variables to denote their distribution, which is an abuse of notation.

Remark: Using Caratheodory's theorem (see e.g. Lemma 3.4 in [2]), one can show that to find $\mathcal{R}_i$, one can restrict one's attention to auxiliary random variables (Q, U_1) that take values in discrete alphabets of sizes $|\mathcal{Q}| \leq 7$ and $|\mathcal{U}_1| \leq |\mathcal{Q}| \cdot |\mathcal{X}_1| + 3$.

Theorem 1: For any IMA channel given by a set of alphabets and a distribution $p_{Y_1,Y_2|X_1,X_2}$, we have

$$\mathcal{R}_i \subseteq \mathcal{R}.$$

To prove Theorem 1, we show the existence of a code that uses superposition at Transmitter 1. The auxiliary random variable U_1, which can be referred to as the cloud center, facilitates the decoding at Receiver 2. The proof details are given in Section V. Note that for structured IMA channels, using the properties of the functions f_j and the fact that S_j depends only on X_j, for $j = 1, 2$, the inequalities of Definition 3 become

$$\begin{aligned} R_1 \leq& H(Y_1|X_2, Q) - H(S_2|X_2, Q), && (7)\\ R_2 \leq& H(S_2|Q) - H(S_2|X_2, Q), && (8)\\ R_1 + R_2 \leq& H(Y_1|Q) - H(S_2|X_2, Q), && (9)\\ R_2 \leq& H(Y_2|U_1, Q) - H(S_1|U_1, Q), && (10)\\ R_1 + R_2 \leq& H(Y_2|Q) - H(S_1|U_1, Q) \\ &+ H(Y_1|U_1, X_2, Q) - H(S_2|X_2, Q), && (11)\\ R_1 + 2R_2 \leq& H(Y_2|Q) - H(S_1|U_1, Q) \\ &+ H(Y_1|U_1, Q) - H(S_2|X_2, Q). && (12) \end{aligned}$$

Note that the random variable Q models the fact that the users can agree on a time-sharing strategy.

IV. Outer Bound for Structured IMA Channels

Definition 4: For a given distribution $p_Q \ p_{X_1|Q} \ p_{X_2|Q}$, define $\mathcal{R}_o(Q, X_1, X_2)$ as the set of all pairs (R_1, R_2) of non-negative real numbers satisfying the 6 constraints

$$\begin{aligned} R_1 \leq & H(Y_1|X_2,Q) - H(S_2|X_2,Q), \\ R_2 \leq & H(S_2|Q) - H(S_2|X_2,Q), \\ R_1 + R_2 \leq & H(Y_1|Q) - H(S_2|X_2,Q), \\ R_2 \leq & H(Y_2|X_1,Q) - H(S_1|X_1,Q), \\ R_1 + R_2 \leq & H(Y_2|Q) - H(S_1|X_1,Q) \\ & + H(Y_1|U_1',X_2,Q) - H(S_2|X_2,Q), \\ R_1 + 2R_2 \leq & H(Y_2|Q) - H(S_1|X_1,Q) \\ & + H(Y_1|U_1',Q) - H(S_2|X_2,Q), \end{aligned} \tag{13}$$

where the auxiliary random variable U_1' takes values in $\mathcal{S}_1$ and follows the distribution

$$p_{U_1'|Q,X_1,X_2,Y_1,Y_2}(u_1|q,x_1,x_2,y_1,y_2) = p_{S_1|X_1}(u_1|x_1),$$

i.e., U_1' is a conditionally independent copy of S_1. In addition, define

$$\mathcal{R}_o \triangleq \cup_{Q,X_1,X_2} \mathcal{R}_o(Q,X_1,X_2),$$

where the union is over all distributions of the form $p_Q\ p_{X_1|Q}\ p_{X_2|Q}$.

Theorem 2: For any structured IMA channel given by a set of alphabets, distributions $p_{S_1|X_1}$, $p_{S_2|X_2}$, and functions f_1, f_2, we have

$$\mathcal{R} \subseteq \mathcal{R}_o.$$

The proof of Theorem 2 can be found in Section VI.

Theorem 3: If $(R_1,R_2) \in \mathcal{R}_o(Q,X_1,X_2)$, then

$$\big(R_1, R_2 - I(X_1;S_1|U_1',Q)\big) \in \mathcal{R}_i(Q,X_1,X_2,U_1') \subseteq \mathcal{R}_i,$$

where U_1' is defined as in Definition 4.

Proof: Let $(R_1,R_2) \in \mathcal{R}_o(Q,X_1,X_2)$. Let the region $\tilde{\mathcal{R}}_o(Q,X_1,X_2)$ be the slight modification of $\mathcal{R}_o(Q,X_1,X_2)$ where we replace the term $H(Y_2|X_1,Q)$ in (13) by $H(Y_2|U_1',Q)$. Since by doing so we relax that bound, (R_1,R_2) is also in $\tilde{\mathcal{R}}_o(Q,X_1,X_2)$. After noticing that $I(X_1;S_1|U_1',Q) = H(S_1|U_1',Q) - H(S_1|X_1,Q)$, one can then verify that the pair $(R_1, R_2 - I(X_1;S_1|U_1',Q))$ satisfies all the inequalities (7) through (12) for $U_1 = U_1'$. ■

Corollary 1: If the channel $p_{S_1|X_1}$ is deterministic, then $\mathcal{R}_i = \mathcal{R}_o = \mathcal{R}$.

Proof: If $p_{S_1|X_1}$ is deterministic, then $U_1' = S_1$, and the gap $I(X_1;S_1|U_1',Q)$ is zero. Thus, we obtain an exact characterization of the capacity region for the structured IMA channel with a deterministic X_1–S_1 channel. ■

Consider a Gaussian IMA channel, where all the alphabets are the complex plane and $S_j = b_jX_j + Z_j$ for $j = 1,2$, where b_j is a complex constant and Z_j is complex, circularly symmetric Gaussian noise independent of (X_j,Q). Let $f_1(X_1,S_2) = a_1X_1 + S_2$, and analogously for $f_2(X_2,S_1)$. The resulting channel can be summarized by the equations

$$\begin{aligned} Y_1 &= a_1X_1 + b_2X_2 + Z_2 \\ Y_2 &= b_1X_1 + a_2X_2 + Z_1. \end{aligned}$$

The following is a second corollary to Theorem 3.

Corollary 2: For the Gaussian IMA channel, $\mathcal{R}_i$ gives a 1-bit approximation of the capacity region.

Proof: Theorems 1 and 2 can be extended to channels with continuous alphabets. In this case $U_1' = b_1X_1 + Z_1'$, where $Z_1' \sim Z_1$ and $Z_1' \perp (X_1,Z_1,Q)$. Denoting by $h(\cdot)$ the differential entropy, we have

$$\begin{aligned} I(X_1;S_1|U_1',Q) &= h(S_1|U_1',Q) - h(S_1|U_1',X_1,Q) \\ &= h(S_1|U_1',Q) - h(Z_1|Q) \\ &\leq h(S_1 - U_1'|Q) - h(Z_1|Q) \\ &= h(Z_1 - Z_1'|Q) - h(Z_1|Q) \\ &= h(Z_1 - Z_1') - h(Z_1) \\ &= \log(2) = 1 \text{ bit}. \end{aligned}$$

■

For the high SNR regime, the approximation given in Corollary 2 can be very useful.

V. Proof of Theorem 1

Assume that $(R_1,R_2) \in \mathcal{R}_i(Q,X_1,X_2,U_1)$ for a given joint distribution $p_Q\ p_{X_1|Q}\ p_{X_2|Q}\ p_{U_1|X_1,Q}$. The random variable Q is a time-sharing variable and is assumed to be available at both transmitters and both receivers, modeling a shared pseudo-random number generator.

Through a random coding argument, we show the existence of a code that achieves rates (R_1,R_2). First, generate a sequence $\mathbf{q}$ from $p_Q(\cdot)^n$. Choose some auxiliary rate $B \in [0,R_1]$. Randomly generate a code in the following way. Generate 2^{nB} sequences $\mathbf{u}_1(i)$ from $\prod_{t=1}^n p_{U_1|Q}(\cdot|q_t)$, and index them by $i \in \{1,\ldots,2^{nB}\}$. For each i, generate $2^{n(R_1-B)}$ sequences $\mathbf{x}_1(i,j)$ from $\prod_{t=1}^n p_{X_1|U_1,Q}(\cdot|u_{1,t}(i),q_t)$, and index them by $j \in \{1,\ldots,2^{n(R_1-B)}\}$. Finally, generate 2^{nR_2} sequences $\mathbf{x}_2(k)$ from $\prod_{t=1}^n p_{X_2|Q}(\cdot|q_t)$, and index them by $k \in \{1,\ldots,2^{nR_2}\}$.

Once this code is generated, it is fixed for all time and used to encode a message pair $(W_1,W_2) = \big((i,j),k\big)$ into transmit vectors $\mathbf{x}_1(i,j)$ and $\mathbf{x}_2(k)$. Receiver 1 declares its estimate $(\hat{W}_1,\hat{W}_2)$ to be the unique triple $\big((i,j),k\big)$ for which $\big(\mathbf{x}_1(i,j),\mathbf{u}_1(i),\mathbf{x}_2(k),\mathbf{y}_1,\mathbf{q}\big) \in T_\delta(X_1,U_1,X_2,Y_1,Q)$, where T_δ is the set of all jointly typical sequences as defined in [2]. If such a unique triple cannot be found, Receiver 1 declares an error. Receiver 2 declares its estimate $\tilde{W}_2$ to be the unique index k for which there exists at least one index i such that $\big(\mathbf{u}_1(i),\mathbf{x}_2(k),\mathbf{y}_2,\mathbf{q}\big) \in T_\delta(U_1,X_2,Y_2,Q)$. If such a unique index cannot be found, Receiver 2 declares an error. An error occurs when a receiver declares an error, or when $(\hat{W}_1,\hat{W}_2) \neq (W_1,W_2)$ or $\tilde{W}_2 \neq W_2$.

Without loss of generality, assume that $(W_1,W_2) = \big((1,1),1\big)$. As $\big(\mathbf{x}_1(1,1),\mathbf{u}_1(1),\mathbf{x}_2(1),\mathbf{y}_1,\mathbf{y}_2,\mathbf{q}\big) \in T_\delta(X_1,U_1,X_2,Y_1,Y_2,Q)$ with high probability, to upper bound the probability of error it is sufficient to consider the events where typicality holds for an index triple $(i,j,k) \neq (1,1,1)$. The error events at Receiver 1 are that $\big(\mathbf{x}_1(i,j),\mathbf{u}_1(i),\mathbf{x}_2(k),\mathbf{y}_1,\mathbf{q}\big) \in T_\delta(X_1,U_1,X_2,Y_1,Q)$ for

- $i \neq 1$, j arbitrary and $k = 1$,
- $i = 1$, $j \neq 1$ and $k = 1$,
- $i = 1$, $j = 1$ and $k \neq 1$,
- $i \neq 1$, j arbitrary and $k \neq 1$,
- $i = 1$, $j \neq 1$ and $k \neq 1$.

The error events at Receiver 2 are that $\big(\mathbf{u}_1(i), \mathbf{x}_2(k), \mathbf{y}_2, \mathbf{q}\big) \in T_\delta(U_1, X_2, Y_2, Q)$ for

- $i \neq 1$ and $k \neq 1$,
- $i = 1$ and $k \neq 1$.

From standard arguments [2] it follows that the expected probability of all these events (where the expectation is over all random codes and over all messages (W_1, W_2)) can be made arbitrarily small if

$$B + (R_1 - B) = R_1 \leq I(X_1; Y_1 | X_2, Q), \tag{14}$$

$$R_1 - B \leq I(X_1; Y_1 | U_1, X_2, Q), \tag{15}$$

$$R_2 \leq I(X_2; Y_1 | X_1, Q), \tag{16}$$

$$B + (R_1 - B) + R_2 = R_1 + R_2 \leq I(X_1, X_2; Y_1 | Q), \tag{17}$$

$$(R_1 - B) + R_2 \leq I(X_1, X_2; Y_1 | U_1, Q), \tag{18}$$

and

$$B + R_2 \leq I(U_1, X_2; Y_2 | Q), \tag{19}$$

$$R_2 \leq I(X_2; Y_2 | U_1, Q). \tag{20}$$

In (14), (16) and (17), we used the Markov chain $U_1 \multimap X_1 \multimap (X_2, Y_1, Y_2)$ (conditioned on Q) to drop U_1 from the right hand side. The constraint $B \in [0, R_1]$ can be written as

$$B - R_1 \leq 0, \tag{21}$$

$$-B \leq 0. \tag{22}$$

Out of the constraints (14) through (22), three are lower bounds on B (namely (15), (18) and (22)), whereas two are upper bounds on B ((19) and (21)). It is clear that an auxiliary rate B that satisfies these 5 bounds exists if and only if every upper bound on B is larger than (or equal to) every lower bound on B. For instance, to be able to find a B that satisfies (15) and (19), we require

$$R_1 + R_2 \leq I(X_1; Y_1 | U_1, X_2, Q) + I(U_1, X_2; Y_2 | Q), \tag{23}$$

and, on the other hand, if such a B exists, than (23) is true. Note that (23) is the same condition as (5) in the claim of the theorem. Analoguously, by combining (18) with (19), we obtain (6). All the other combinations of lower and upper bounds on B yield inequalities that are already implied by (14), (16), (17), (20), or by the non-negativity of the rate R_1 or the mutual information. Hence, inequalities (14) through (22) are true for some B if and only if (1) through (6) are true (together with $R_1, R_2 \geq 0$). But these 6 inequalities hold since we assume that $(R_1, R_2) \in \mathcal{R}_i(Q, X_1, X_2, U_1)$. Thus, the expected probability of all error events can be made arbitrarily small by choosing n large enough. It follows that there exists at least one code for which all decoding error probabilities are arbitrarily small.

VI. Proof of Theorem 2

The proof follows the same lines as the proof of Theorem 1 in [7]. Assume that there exists an IMA channel code of blocklength n which achieves a small error probability for a structured channel given by $p_{S_j|X_j}$ and f_j for $j = 1, 2$ and the corresponding alphabets. Let W_1 and W_2 be independent messages, uniformly distributed in $\{1, \ldots, 2^{nR_1}\}$ and $\{1, \ldots, 2^{nR_2}\}$, and let $\mathbf{X}_1$, $\mathbf{X}_2$, $\mathbf{S}_1$, $\mathbf{S}_2$, $\mathbf{Y}_1$, $\mathbf{Y}_2$ be the random n-sequences induced by them, the encoders and the channel. Generate a sequence $\mathbf{U}_1'$ by passing $\mathbf{X}_1$ through an auxiliary channel described by $p_{S_1|X_1}$. Note that, by construction, $\mathbf{U}_1' \multimap \mathbf{X}_1 \multimap (\mathbf{X}_2, \mathbf{S}_1, \mathbf{S}_2, \mathbf{Y}_1, \mathbf{Y}_2)$ forms a Markov chain, and $(\mathbf{U}_1, \mathbf{X}_1)$ has the same distribution as $(\mathbf{S}_1, \mathbf{X}_1)$. In particular, $H(\mathbf{U}_1) = H(\mathbf{S}_1)$ and $H(\mathbf{U}_1|\mathbf{X}_1) = H(\mathbf{S}_1|\mathbf{X}_1)$.

By Fano's inequality we have

$$nR_j \leq I(\mathbf{X}_j; \mathbf{Y}_j) + n\epsilon_n,$$

for $j = 1, 2$,

$$nR_2 \leq I(\mathbf{X}_2; \mathbf{Y}_1) + n\epsilon_n,$$

and

$$n(R_1 + R_2) \leq I(\mathbf{X}_1, \mathbf{X}_2; \mathbf{Y}_1) + n\epsilon_n.$$

We can then write

$$\begin{aligned} n(R_1 - \epsilon_n) &\leq I(\mathbf{X}_1; \mathbf{Y}_1) \\ &\leq I(\mathbf{X}_1; \mathbf{Y}_1, \mathbf{X}_2) \\ &= I(\mathbf{X}_1; \mathbf{Y}_1 | \mathbf{X}_2) \\ &= H(\mathbf{Y}_1|\mathbf{X}_2) - H(\mathbf{Y}_1|\mathbf{X}_1, \mathbf{X}_2) \\ &= H(\mathbf{Y}_1|\mathbf{X}_2) - H(\mathbf{S}_2|\mathbf{X}_2) \\ &\leq \sum_{t=1}^{n} \big[H(Y_{1,t}|X_{2,t}) - H(S_{2,t}|X_{2,t})\big], \end{aligned} \tag{24}$$

where the last inequality holds because $H(\mathbf{Y}_1|\mathbf{X}_2)$ is upper bounded by its single-letter form, and $H(\mathbf{S}_2|\mathbf{X}_2)$ is equal to its single-letter form, because $\mathbf{S}_2$ is the output of a memoryless channel whose input is $\mathbf{X}_2$. Following the same initial steps, we obtain

$$\begin{aligned} n(R_2 - \epsilon_n) &\leq I(\mathbf{X}_2; \mathbf{Y}_1) \\ &\leq H(\mathbf{Y}_1|\mathbf{X}_1) - H(\mathbf{Y}_1|\mathbf{X}_1, \mathbf{X}_2) \\ &= H(\mathbf{S}_2) - H(\mathbf{S}_2|\mathbf{X}_2) \\ &\leq \sum_{t=1}^{n} \big[H(S_{2,t}) - H(S_{2,t}|X_{2,t})\big]. \end{aligned} \tag{25}$$

In a similar manner, we have

$$\begin{aligned} n(R_1 + R_2 - \epsilon_n) &\leq I(\mathbf{X}_1, \mathbf{X}_2; \mathbf{Y}_1) \\ &= H(\mathbf{Y}_1) - H(\mathbf{S}_2|\mathbf{X}_2) \\ &\leq \sum_{t=1}^{n} \big[H(Y_{1,t}) - H(S_{2,t}|X_{2,t})\big] \end{aligned} \tag{26}$$

and

$$
\begin{aligned}
&n(R_2-\epsilon_n)\\
&\quad \le I(\mathbf{X}_2;\mathbf{Y}_2)\\
&\quad \le I(\mathbf{X}_2;\mathbf{Y}_2,\mathbf{X}_1)\\
&\quad = I(\mathbf{X}_2;\mathbf{Y}_2|\mathbf{X}_1)\\
&\quad = H(\mathbf{Y}_2|\mathbf{X}_1)-H(\mathbf{S}_1|\mathbf{X}_1)\\
&\quad \le \sum_{t=1}^{n}\big[H(Y_{2,t}|X_{1,t})-H(S_{1,t}|X_{1,t})\big]. \qquad (27)
\end{aligned}
$$

We further obtain

$$
\begin{aligned}
&n(R_1{+}R_2-2\epsilon_n)\\
&\quad \le I(\mathbf{X}_1;\mathbf{Y}_1,\mathbf{U}_1',\mathbf{X}_2)+I(\mathbf{X}_2;\mathbf{Y}_2)\\
&\quad = I(\mathbf{X}_1;\mathbf{U}_1')+\underbrace{I(\mathbf{X}_1;\mathbf{X}_2|\mathbf{U}_1')}_{=0}\\
&\qquad + I(\mathbf{X}_1;\mathbf{Y}_1|\mathbf{U}_1',\mathbf{X}_2)+I(\mathbf{X}_2;\mathbf{Y}_2)\\
&\quad = \underbrace{H(\mathbf{U}_1')}_{=H(\mathbf{S}_1)}-\underbrace{H(\mathbf{U}_1'|\mathbf{X}_1)}_{=H(\mathbf{S}_1|\mathbf{X}_1)}+H(\mathbf{Y}_1|\mathbf{U}_1',\mathbf{X}_2)\\
&\qquad - H(\mathbf{S}_2|\mathbf{X}_2)+H(\mathbf{Y}_2)-H(\mathbf{S}_1)\\
&\quad \le \sum_{t=1}^{n}\big[H(Y_{2,t})-H(S_{1,t}|X_{1,t})\\
&\qquad + H(Y_{1,t}|U_{1,t}',X_{2,t})-H(S_{2,t}|X_{2,t})\big], \qquad (28)
\end{aligned}
$$

where the indicated term is zero because W_1 and W_2 are independent. Finally,

$$
\begin{aligned}
&n(R_1{+}2R_2-3\epsilon_n)\\
&\quad \le I(\mathbf{X}_1;\mathbf{Y}_1,\mathbf{U}_1')+I(\mathbf{X}_2;\mathbf{Y}_2)\\
&\qquad + I(\mathbf{X}_2;\mathbf{Y}_1,\mathbf{S}_2)\\
&\quad = I(\mathbf{X}_1;\mathbf{U}_1')+I(\mathbf{X}_1;\mathbf{Y}_1|\mathbf{U}_1')+I(\mathbf{X}_2;\mathbf{Y}_2)\\
&\qquad + I(\mathbf{X}_2;\mathbf{S}_2)+\underbrace{I(\mathbf{X}_2;\mathbf{Y}_1|\mathbf{S}_2)}_{=0}\\
&\quad = \underbrace{H(\mathbf{U}_1')}_{=H(\mathbf{S}_1)}-H(\mathbf{U}_1'|\mathbf{X}_1)+H(\mathbf{Y}_1|\mathbf{U}_1')-H(\mathbf{S}_2)\\
&\qquad + H(\mathbf{Y}_2)-H(\mathbf{S}_1)+H(\mathbf{S}_2)-H(\mathbf{S}_2|\mathbf{X}_2)\\
&\quad = H(\mathbf{Y}_2)-H(\mathbf{U}_1'|\mathbf{X}_1)+H(\mathbf{Y}_1|\mathbf{U}_1')-H(\mathbf{S}_2|\mathbf{X}_2)\\
&\quad \le \sum_{t=1}^{n}\big[H(Y_{2,t})-H(S_{1,t}|X_{1,t})\\
&\qquad + H(Y_{1,t}|U_{1,t}')-H(S_{2,t}|X_{2,t})\big]. \qquad (29)
\end{aligned}
$$

Setting (Q, X_1, X_2) to be random variables with Q uniformly distributed in $\{1,\dots,n\}$ and $p_{X_j|Q}(x_j|t)=\mathbf{P}(X_{j,t}=x_j)$, for $j=1,2$, we see that the inequalities (24) through (29) can be rewritten as $(R_1,R_2)\in\mathcal{R}_o(Q,X_1,X_2)\subseteq\mathcal{R}_o$. As $\mathcal{R}_o$ is closed, we see that any achievable rate pair is in $\mathcal{R}_o$.

VII. Discussion

In this paper, we have introduced the IMA channel coding problem, inspired by multi-user communication over wireless networks. We have proved an inner bound on the capacity region of the general IMA channel. For the structured IMA channel, we have provided an outer bound on the capacity region and quantified the gap between inner and outer bound. For the structured IMA channel with deterministic $p_{S_1|X_1}$, we have found that the gap is zero. This is termed the semi-deterministic IMA channel since $p_{S_2|X_2}$ could still be probabilistic.

Characterizing the exact capacity region for general IMA channels, however, seems as challenging as the general interference channel. Note that since Transmitter 1 is interested in both messages, the IMA channel is not a special case of the interference channel, nor is the opposite the case. Philosophically, the IMA channel is close to a degraded message set problem.

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