Multiuser DMT: A Multiple Access Modulation Scheme

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Abstract

In this paper we propose a multiuser DMT scheme for use in a frequency dispersive multiple access channel. We formulate the problem as an optimization problem for maximizing the rate-sum or a weighted sum of the rates. We establish the rate-sum maximizing solution which is a multiuser waterfilling scheme over the DMT tones. We propose a multiuser bit-loading algorithm which finds the rate-maximizing solution in a finite number of steps. We study coding strategies and examine some implementation issues. Finally we present some numerical results obtained by using this scheme and we draw some conclusions from these results.

1. INTRODUCTION

Recently there has been considerable interest in multiple access communication especially over frequency selective channels. These channels are encountered in a variety of applications such as hybrid fiber coax (HFC) networks, wireless LAN, cellular telephony etc. Most of the current multiple access schemes are single-carrier modulation schemes, though there has been some recent interest [1, 2] in applying multicarrier schemes to Multiple Access Channels (MAC). The attraction to multicarrier schemes is mainly due to simpler receiver structures. Also, input spectral shaping is easily done in multicarrier schemes. In this paper we are concerned with efficient transmission schemes when the channels are *known*. Note that the channels need be known only at the receiver where the bit allocations are calculated and fed back to the transmitters. This could be quite reasonable in applications such as HFC networks and wireless LAN. We also wish to develop a framework to study other multiple access modulation/coding schemes.

The problem of transmitting data over a spectrally shaped channel has been well studied in the context of single user communications [3, 4]. Only recently [5] has the information theoretic capacity region of multiple access channels in frequency dispersive channels been studied. In this work the multiple user capacity was studied by using a parallel channel approach as done in the single user problem. In this paper we establish a framework for using multicarrier modulation schemes in a multiple access channel. In order to create parallel ISI-free subchannels we need to simultaneously diagonalize the channel responses of all the users. In general channel responses are not simultaneously diagonalizable, and hence we have to create channel responses that have the same eigenvectors. Therefore several schemes used in single-user channels [6] do not carry over to this case. Our proposed solution is to use a Discrete Multitone scheme which is channel independent and we can show that it can *simultaneously* decompose all the responses into parallel channels.

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In multiple access communications several different optimization criteria could arise. In this paper we formulate a general optimization criterion, but we mainly study the rate-sum maximizing criterion.

The paper is organized as follows. In Section 2 we describe the transmission and receiver structures. We formulate and solve the rate optimization criteria in Section 3. Section 4 describes a multiuser waterfilling algorithm as a solution to the rate-maximizing problem. We describe some possible coding schemes and some lower complexity methods in Section 5. In Section 6 we present some numerical examples of the scheme described in the paper. Finally we have some concluding remarks in Section 7.

2. TRANSMISSION AND RECEIVER STRUCTURES

In subsection 2.1 we establish the notation for the DMT-based multiple-access scheme. In subsection 2.2 we describe the requirements for the transceiver structure. We also define the receiver structure and describe its differences with the matched filter bank approach.

2.1. NOTATION

A block diagram of a single-user DMT scheme is illustrated in Figure 1. The transmission is assumed to be passband for generality though the scheme is equally applicable to baseband transmission. Spectral shaping of the input bit-stream is done by the bit-allocation scheme. The output $\{X(i)\}_0^{N-1}$ is the frequency domain data symbol. The complex data samples $\{x(l)\}$ are produced by taking the IDFT of the data symbol $\{X(i)\}$ and adding a cyclic prefix (length ν) to create the DFT based parallel channels [3]. The two streams (real and imaginary) are passed through the DAC at a sampling period of $T^{'} = T_{sym}/(N+\nu)$, where T_{sym} is the DMT symbol period. To have no aliasing we ensure that the highest frequency of the baseband signal is no more than $1/(2T^{'})$. The transmit filter would in general be a low-pass filter with the above requirements. Using this we can write the recovered demodulated signal as

$$y(t) = \sum_{l} x(l)h(t - lT^{'}) + z(t) \tag{1}$$

where $h(t) = h_c(t) * g(t)$, and $h_c(t)$ is the complex baseband equivalent of the passband channel impulse response. We can sample the demodulated signal at the Nyquist rate of $1/T^{'}$. After cyclic prefix removal and the DFT of the received packet we obtain:

$$Y(i) = H(i)X(i) + Z(i), i = 0, \dots, N-1 \tag{2}$$

where $\{H(i)\}$ is the DFT of the $h[k] = h(kT^{'})$. We have assumed that the impulse response is of finite duration which is quite reasonable in many applications. The above development of notation was done in the single user case. If several users are synchronized we can obtain a multiple access DMT notation by adding the responses of the U users as,

$$Y(i) = \sum_{u=1}^{U} H_u(i)X_u(i) + Z(i) \tag{3}$$

Here the prefix length is determined by the longest channel impulse response. We assume that $\{Z(i)\}$ are Gaussian and independent with powers $\{N(i)\}$. Note that this model could be extended to the case when the transmitters and receivers have several sensors. This is denoted as,

$$\mathbf{Y}(i) = \sum_{u=1}^{U} \mathbf{H}_u(i)\mathbf{X}_u(i) + \mathbf{Z}(i) \tag{4}$$

where the transmit and received signals at each frequency is a vector (representing the multiple sensors).

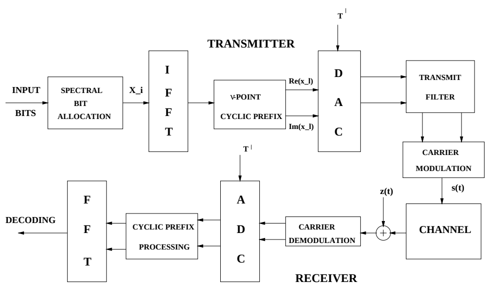

Figure 1: A single user DMT system.

2.2. REQUIREMENTS

The transmission structure described above assumed symbol alignment between all the users. To avoid unnecessarily long prefix lengths the users advance transmission by the appropriate transmission delays so as to arrive synchronously. The issue of completely asynchronous transmission is considered in Section 5. We assume that the N subcarriers represent frequencies from $-1/(2T^{'})$ to $1/(2T^{'})$. We have $N+\nu$ data samples every DMT symbol period (T_{sym}). Therefore we have one complex data sample every $T^{'}$ seconds. At the receiver (after frequency down-conversion) the sufficient statistics are collected by sampling the signal at the Nyquist rate $1/T^{'}$, and we obtain a complex sample at each sample time. Therefore, we have constructed a parallel scalar MAC. This is in contrast to a bank of matched filters which would necessarily create a vector MAC. Due to the transmission scheme we have simplified the receiver structure. This is the model we will study in this paper.

3. RATE OPTIMIZATION

From an information theoretic viewpoint it has been shown recently [5] that the boundary of the MAC rate region can be obtained by waterfilling across several users. Their parallel channel approach indicates that a DMT is an asymptotically optimal transceiver structure. This naturally extends the role of DMT to the multiuser environment.

In a single user environment, we maximize the transmission rate for a given performance level. In a multiuser environment we could have several optimization criteria. For example, we may choose to maximize the rate-sum of the users, or a weighted rate-sum chosen as our criterion. The weights are interpreted as a measure of priority of the user. Therefore we can write the general problem as,

$$\begin{aligned} &\max_{\{R_u\}\in\mathcal{R}} \quad \textstyle\sum_u \gamma_u R_u \\ &s.t. \mathbb{E}(|X_u|^2) < P_u, \quad u = 1, \dots, U \\ &P_e(u) < P_{e,min} \quad u = 1, \dots, U \end{aligned} \tag{5}$$

where $\sum \gamma_u = 1$. For simplicity of exposition, we will concentrate mostly on the two user case in the sequel. Also in this paper we will concentrate on the rate-sum maximization problem, which corresponds to $\gamma_u = 1/U \; \forall u$.

The issue of finding the probability of error for a MAC is a hard problem. It is highly dependent on the transmission and detection scheme. To be able to tackle this problem in a simple manner, we make an approximation. In single-user schemes a single parameter characterization called the SNR gap [7] is possible. This characterizes the probability of error for a large range of coding rates and a class of coding schemes. If we view the decoding of a MAC in terms of single user decoders we could use the above characterization. Looking ahead to the solution of the rate-sum maximization, it turns out that the subchannels are partitioned among users. In this case the single user gap characterization applies directly. Therefore we can write the achievable rate-sum as,

$$R = \sum_{i=0}^{N-1} \frac{1}{2} log(1 + \sum_{u=1}^{U} \frac{|H_u(i)|^2 S_u(i)}{\Gamma(\mathcal{C})N(i)}) \tag{6}$$

where $\Gamma(\mathcal{C})$ represents the single user gap using the coding scheme $\mathcal{C}$ for the given probability of error.

However, under certain special conditions the solution could require the subchannels to be shared. This occurs less frequently and we can justify the criterion in (6) based on successive interference cancellation [8]. We defer the details of this argument to [9]. Note in passing that the general solution to the weighted rate-sum maximization causes subchannels to be shared. With this caveat in mind we proceed to use (6) as the quantity to be maximized. Thus using the gap approximation we can get a relationship between the number of bits and constellation expansion factors needed in the QAM constellation to be used.

Thus, we can state the rate-sum maximization problem as follows:

$$\begin{aligned} &\max_{\{S_u(i)\}} \sum_{i=0}^{N-1} \frac{1}{2} log(1 + \sum_{u=1}^{U} \frac{|H_u(i)|^2 S_u(i)}{\Gamma(\mathcal{C})N(i)}) \\ &s.t. \frac{1}{N} \sum_i S_u(i) < P_u, u = 1, \dots, U \end{aligned} \tag{7}$$

where the gap $\Gamma(\mathcal{C})$ ensures the $P_e(u) \leq P_{e,min}$. As mentioned earlier we will restrict our attention mostly to the two-user case. We can solve this optimization problem using Kuhn-Tucker conditions. This formulation is similar to that in [5] and we can state the following result:

$$\lambda_1 S_1(i) + \lambda_2 S_2(i) = [1 - min(\lambda_1 T_1^{-1}(i), \lambda_2 T_2^{-1}(i))]^+ \quad (8)$$

$$\text{C1} : \lambda_1 S_1(i) = [1 - \lambda_1 T_1^{-1}(i)]^+, S_2(i) = 0,$$

$$\text{C2} : \lambda_2 S_2(i) = [1 - \lambda_2 T_2^{-1}(i)]^+, S_1(i) = 0,$$

$$\text{C3} : 1 + \lambda_1 S_1(i) + \lambda_2 S_2(i) = \lambda_1 T_1^{-1}(i) = \lambda_2 T_2^{-1}(i)$$

where $T_k(i) = \frac{|H_k(i)|^2}{\Gamma(\mathcal{C})N(i)}$, $k = 1, 2$, $C1 : \frac{\lambda_1}{|H_1(i)|^2} < \frac{\lambda_2}{|H_2(i)|^2}$, $C2 : \frac{\lambda_1}{||H_1(i)||^2} > \frac{\lambda_2}{||H_2(i)||^2}$ and $C3 : \frac{\lambda_1}{||H_1(i)||^2} = \frac{\lambda_2}{||H_2(i)||^2}$ As mentioned earlier this shows that we could have subchannels shared between both users. Clearly the solution as stated in (8) is an implicit solution and we need to develop an efficient numerical technique to solve this problem. This is the extension of the classical waterfilling problem to the multiple user case.

4. A MULTIUSER WATERFILLING SCHEME

In this section we briefly describe a multiuser bit-loading algorithm which converges in a *finite* number of steps to the rate-maximizing multiuser waterfilling solution. We describe a two-user scheme for simplicity and indicate methods to extend this to the general case. In subsection 4.1 we describe the two-user waterfilling algorithm. In subsection 4.2 we indicate methods to extend this to the general U-user case.

4.1. TWO USER WATERFILLING

In the two user scenario we need to determine the Lagrange multipliers λ_1 and λ_2 in (8) Let us define, $\Delta_s(\lambda_1, \lambda_2) = \sum_{i=0}^{N-1}[1 - min(\lambda_1 T_1^{-1}(i), \lambda_2 T_2^{-1}(i)]^+ - (\lambda_1 P_1 + \lambda_2 P_2)$ and $\Delta_1(\lambda_1, \lambda_2) = \sum_{i \in I_1}[1 - \lambda_1 T_1^{-1}(i)]^+ - \lambda_1 P_1$ where, $I_1 = \{i : \frac{\lambda_1}{||H_1(i)||^2} < \frac{\lambda_2}{||H_2(i)||^2}\}$. The following facts would be useful in developing the algorithm. (1)$\Delta_s(\lambda_1, \lambda_2)$ is a strictly decreasing function of λ_1 and λ_2. (2)$\Delta_1(\lambda_1, \lambda_2)$ is a strictly decreasing function of λ_1 for a given λ_2, and is an increasing function of λ_2 for a given λ_1. (3)$\Delta_s(\lambda_1, \lambda_2)$ and $\Delta_1(\lambda_1, \lambda_2)$ are piecewise linear functions of λ_1 and λ_2.

Lemma 4.1 *The curve $\Delta_s(\lambda_1, \lambda_2) = 0$ has a negative slope in the $\lambda_1 - \lambda_2$ plane.*

Lemma 4.2 *The number of allowable partitions I_1 solving (8) is N.*

We have left out the proof for brevity and the details can be found in [9]. The basic idea of the algorithm is illustrated pictorially in Figure 2. The goal is to find $(\lambda_1^*, \lambda_2^*)$ such that it satisfies (8). Therefore, it needs to satisfy $\Delta_s(\cdot) = 0$ and $\Delta_1(\cdot) = 0$ simultaneously. In Figure 2 we have drawn the curve $\Delta_s(\cdot) = 0$ and also drawn lines corresponding to $\frac{\lambda_1(k)}{\lambda_2(k)} = \frac{|H_1(i_k)|^2}{|H_2(i_k)|^2}$. Here $\{i_k\}$ represents the sorted index of $\{\frac{|H_1(i)|^2}{|H_2(i)|^2}\}$. Therefore the segments between the lines correspond to different partitions. The idea is to traverse the curve $\Delta_s(\cdot) = 0$ stepping through $(\lambda_1(k), \lambda_2(k))$ points and evaluating $\Delta_1(\lambda_1(k), \lambda_2(k))$. If $\Delta_1(\cdot) > 0$ we realize from the stated facts that we need $\lambda_1 \uparrow, \lambda_2 \downarrow$. Similarly for $\Delta_1(\cdot) < 0$ we need $\lambda_1 \downarrow, \lambda_2 \uparrow$. The negative slope of the curve $\Delta_s(\cdot) = 0$ (Lemma 4.1) allows us to traverse it satisfying the above requirements. Note that when we detect a sign change in $\Delta_1(\cdot)$ we have found the right segment, and hence the right partition. Using this partition we can find $(\lambda_1^*, \lambda_2^*)$. Note that if the equality condition is satisfied then, we have a sign change between $\Delta_1(\lambda_1(k), \lambda_2(k))$ and $\Delta_1(\lambda_1(k) + \delta, \lambda_2(k))$, where $\delta = sign(\Delta_1(\cdot))\epsilon$ and $\epsilon > 0$ is a small number.

Therefore, we have to search through at most N partitions to reach the solution. Once the partition has been determined it takes O(N) steps to reach the solution. However, to set up the partitions it takes $O(N^2)$ steps as it takes N steps to determine $(\lambda_1(k), \lambda_2(k))$ for each partition. Hence we have a scheme that finds the solution after $O(N^2)$ steps of computation. Therefore,

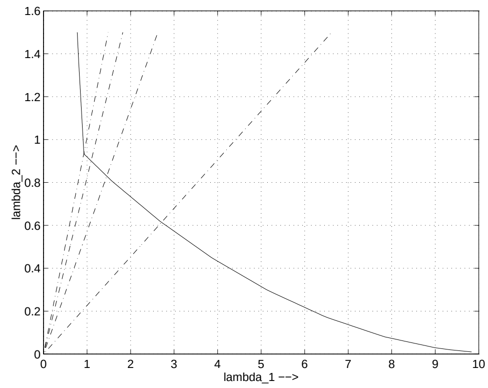

Figure 2: Description of Rate Maximizing Solution.

using all these stated facts we put together the following algorithm:

1. Find $(\lambda_1^{(l)}, \lambda_2^{(l)})$ s.t. $\Delta_s(\cdot) = 0$ and $\frac{\lambda_1^{(l)}}{\lambda_2^{(l)}} = \frac{|H_1(i_l)|^2}{|H_2(i_l)|^2}$ for $l = 0, \ldots, N-1$. Let $k = 1$.
2. If $\Delta_1(\lambda_1^{(k)}, \lambda_2^{(k)}) = 0$ then we have found the solution and go to Step 8.
3. Otherwise if $\Delta_1(\lambda_1^{(k)}, \lambda_2^{(k)}) > 0$ we step to the next partition such that $\lambda_2^{(k+1)} < \lambda_2^{(k)}$. else if $\Delta_1(\lambda_1^{(k)}, \lambda_2^{(k)}) < 0$ we step to the next partition such that $\lambda_2^{(k+1)} > \lambda_2^{(k)}$.
4. If sign$(\Delta_1(\lambda_1^{(k)}, \lambda_2^{(k)})) \neq$ sign$(\Delta_1(\lambda_1^{(k-1)}, \lambda_2^{(k-1)}))$ then we have found the optimal partition.
5. If the optimum partition is not found set $k = k + 1$ and go to step 2.
6. If the optimum partition is found check if the optimal λ_1, λ_2 is such that $\frac{\lambda_1}{\lambda_2} = \frac{|H_1(i_k)|^2}{|H_2(i_k)|^2}$. If this is the case we need to use the equality conditions shown in (8).
7. If equality conditions in Step 7 are not satisfied we find the optimal λ_1^*, λ_2^* by using the partition found in Step 5 and the piecewise linear condition.
8. The solution is found and we find the optimal rate region.

4.2. GENERAL CASE

For a general multiuser case we iterate over $\lambda_1, \ldots, \lambda_U$. In the U user case $N^{U-1} \leq P \leq N^{U(U-1)/2}$, where P is the number of partitions. The algorithm steps through each of the partitions and therefore in the worst case would have complexity $N^{U(U-1)/2+1}$. This is exponential in the number of users and would be significant for a large number of users. Therefore there is a need to look for low-complexity algorithms and this is the topic of Section 5.

5. PRACTICAL CONSIDERATIONS

In the previous sections we have outlined an efficient transceiver structure suitable for a multipoint-to-point communications in dispersive channels. We described a joint water-filling algorithm which maximizes the sum of rates. In this section we consider practical issues associated with the multiuser DMT scheme. In subsection 5.1 we describe encoder and decoder schemes to be used with the multiuser DMT scheme. In subsection 5.2 we delve into issues such as complexity and scalability associated with this scheme.

5.1. ENCODER AND DECODER STRUCTURES

As the user codebooks are chosen independently, once the bit assignments (using the waterfilling algorithm) is complete, each user can independently design the codes for the particular transmit spectrum. In order to ensure a given probability of error the codes need to be chosen with the required coding gain. This is an easy task when the optimal joint spectrum is such that each subchannel has only one user. This is typically true except for certain special cases (Section 3) where subchannel sharing becomes necessary. We concentrate on the first case in this paper and the details of the special case are described in [9].

In the typical solution each subchannel has a single user. Each user codes over the set of subchannels it has been assigned. The codes are single user codes and we can easily ensure the required probability of error by designing a sufficiently powerful trellis coding scheme. The issues of concatenating trellis codes with multicarrier schemes have been studied extensively in literature [3, 10]. The decoder is quite simple - it separates the subchannels used by each user and decodes each user independently. Hence all the issues of single user encoding and decoding apply directly here. For latency and complexity reasons it is advantageous to code across the used subchannels instead of parallel independent coders [10].

5.2. IMPLEMENTATION CONSIDERATIONS

The multiuser waterfilling algorithm described in Section 4 has complexity which is exponential in U. This could become quite significant for large scale systems. Moreover the algorithm is not easily scalable *i.e.* if a new user is added the power spectra of *all* the users need to be recomputed. This could potentially be quite important in random access channels where the number of users using the medium could vary drastically with time. Also we had imposed symbol synchronism on the transmission structure, and we consider schemes that could be asynchronous. Hence we consider a few (albeit suboptimal) approaches to solving this problem.

The first scheme is a TDMA approach where the users time-share the channel. The rates achievable by this scheme is dictated by the line AB shown in Figure 3. When the channels are very disimilar the TDMA approach could be quite suboptimal. However the simplicity of this scheme is very attractive. New users can be added on to unused time slots. Moreover there have been many random access strategies (such as slotted ALOHA) based on the time-sharing principle. Also, if the channels are very similar then the TDMA schemes could be quite close to rate-sum optimal for high SNRs.

Another scheme that could be used is that of multicarrier CDMA adapted to this scenario. Each user spreads the signal bandwidth using orthogonal spreading codes. In effect the users are trying to create very disimilar channels. Here again scalability is easily incorporated and random access strategies (such as CDMA-ALOHA) can also be incorporated. Another advantage of this scheme is that the users do not need to have symbol synchronism. Though this scheme is suboptimal it does utilize the differences in the channels, which help reduce the multiple access interference and thus aid performance.

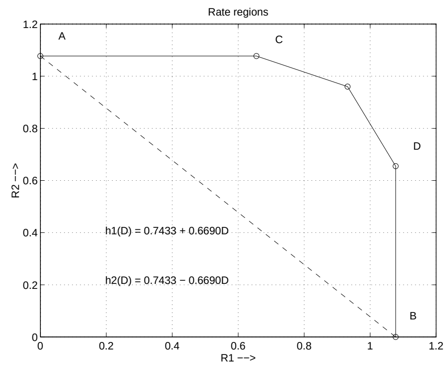

Figure 3: Rate region.

We propose another encoding and decoding strategy based on the insights gained from the framework developed. We notice that the rate-sum points of C and D in Figure 3 are quite close to optimal. There is a simple way of achieving these rate points using an onion peeling approach. We realize from the rate region in Figure 3 that even when one of the users is transmitting at its full rate, we can accommodate other users at a lower rate. If user 2 is transmitting at full rate, we can achieve point C in Figure 3 by transmitting user 1 at a rate which is the waterfilling solution to a modified noise spectrum corresponding to the addition of user 2 and the noise. Therefore we are in effect waterfilling on top of user 2. Thus the user with the highest priority does the waterfilling as if the no other user were present. Next, the user with the next highest priority does waterfilling on top of the first user and noise and so on. This proceeds iteratively till the last user. The advantage of this scheme is that a new user can be given the lowest priority and waterfills according to the spectrum determined by the other users. Hence the encoders transmit using a "successive waterfilling" scheme. The decoding proceeds by decoding the user with the lowest priority first and then successively peeling layers to iteratively decode the users [8]. Due to this structure the successive waterfilling scheme does not need symbol synchronism among the users. This scheme has an encoding complexity of O(NU) and it is attractive as it comes quite close to the joint optimal scheme.

Finally we touch upon issues when there are multiple transmitting and receiving sensors. This scenario has attracted a lot of attention recently, in the context of spatio-temporal processing. The rate-sum maximization of this problem also leads to a multiple-user waterfilling over the users and the spatial dimensions represented by the sensors [5]. Among the most important advantages of using multiple sensors is that of signal separability. This is the basic idea behind the so-called Space-Division-Multiple-Access (SDMA) schemes. It should be noted that the increase of dimensionality (*i.e.* the number of sensors) helps in signal separability and the rate regions then tend to be more rectangular. Therefore the multiple sensors also help at the encoders allowing for almost independent encoding strategies (instead of tightly coupled joint optimizations). The issues related to the increase in signal separability with dimension is discussed in [9].

6. NUMERICAL EXAMPLES

In this section we briefly describe some of the numerical results concerning rate regions and the maximum rate solution for certain example channels.

In Figure 4 we assume a 5dB coding gain and an operation at an error rate of 10^{-6}. We notice the region is close to a pentagon when the channels are similar. In this case we would expect TDMA to perform quite well at high SNRs. Notice that in both cases the successive waterfilling scheme is very close to rate-sum optimality.

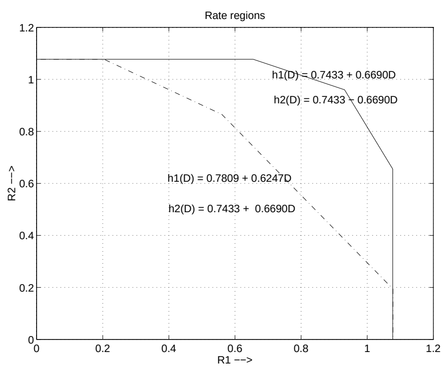

Figure 4: Rates for different channel pairs.

Next Figure 5 illustrates the rate difference (in bits) between TDMA and the joint optimal scheme. In this case again we used codes with a 5dB coding gain and operated at an error rate 10^{-6}. The curves 1 in Figure 5 corresponded to the channel pair $h_1(D) = (1 + 0.8D)/||h_1||$ and $h_2(D) = (1 + 0.9D)/||h_2||$. The curves 2 corresponded to $h_1(D) = (1 + 0.9D)/||h_1||$ and $h_2(D) = (1 - 0.9D)/||h_2||$. These are the same channels used in Figure 4. Note that we could come quite close to the optimal solution by using the successive waterfilling scheme. Again for high SNRs when the channels are similar the TDMA scheme does come close to optimal.

7. CONCLUSIONS

In this paper we have attempted to develop a framework to study multiple access modulation schemes in channels with ISI.

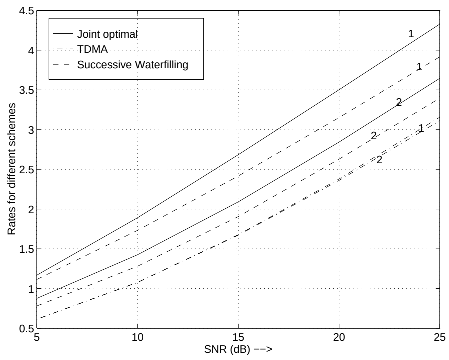

Figure 5: Maximum Rate sum.

We started with the problem formulation where we described the optimization criteria that could be used. Next, we described a DMT-based solution which we can show is an asymptotically optimal structure. We proceeded to develop a multiuser bit-loading algorithm which has a guaranteed convergence in a finite number of steps. We looked into some details of the coding schemes needed to implement this scheme and mentioned some pragmatic strategies. Finally we illustrated some of the points made in this paper through some numerical examples calculated using the bit-loading algorithm developed.

8. REFERENCES

[1] S. Kondo and L. B. Milstein, "Performance of Multicarrier DS CDMA Systems," *IEEE Transacations on Communications*, vol. 44, pp. 238–246, February 1996.

[2] K. S. Jacobsen, J. Bingham, and J. Cioffi, "Synchronized DMT for multipoint-to-point Communications on HFC Networks," in *IEEE GLOBECOM*, pp. 963–966, 1995.

[3] A. Ruiz, J. M. Cioffi, and S. Kasturia, "Discrete Multiple Tone Modulation with Coset Coding for the Spectrally Shaped Channel," *IEEE Transactions on Communications*, vol. 40, pp. 1012–1029, June 1992.

[4] M. V. Eyuboglu and G. D.Forney, "Trellis precoding: combined coding, precoding and shaping for intersymbol interference channels," *IEEE Transactions on Information Theory*, vol. 38, pp. 301–314, March 1992.

[5] R. S. Cheng and S. Verdu, "Gaussian Multiaccess Channels with ISI: Capacity Region and Multiuser Water-filling," *IEEE Transactions on Information Theory*, vol. 39, pp. 773–785, May 1993.

[6] S. Kasturia, J. Aslanis, and J. M. Cioffi, "Vector coding for partial response channels," *IEEE Transactions on Information Theory*, vol. 36, pp. 741–762, July 1989.

[7] J. Cioffi, G.P.Dudevoir, V. Eyuboglu, and G. Forney, "MMSE decision-feedback equalizers and coding. II. Coding results," *IEEE Transactions on Communications*, vol. 43, pp. 2595–2604, October 1995.

[8] A. J. Viterbi, "Very low rate convolution codes for maximum theoretical performance of spread-spectrum multiple-access channels," *IEEE Journal on Selected Areas in Communications*, vol. 8, pp. 641–649, May 1990.

[9] S. N. Diggavi, "Multiuser DMT: A Multiple Access Modulation scheme." in preparation, 1996.

[10] J. C. Tu, *Theory, Design and Application of Multichannel Modulation for Digital Communications*. PhD thesis, Stanford University, Stanford, CA., 1991.