On the Sum-Capacity with Successive Decoding in Interference Channels

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***Abstract*—In this paper, we investigate the sum-capacity of the two-user Gaussian interference channel with Gaussian superposition coding and successive decoding. We first examine an approximate deterministic formulation of the problem, and introduce the complementarity conditions that capture the use of Gaussian coding and successive decoding. In the deterministic channel problem, we show that the constrained sum-capacity *oscillates* as a function of the cross link gain parameters between the information theoretic sum-capacity and the sum-capacity with interference treated as noise. Furthermore, we show that if the number of messages of either user is fewer than the minimum number required to achieve the constrained sum-capacity, the maximum achievable sum-rate drops to that with interference treated as noise. We translate the optimal schemes in the deterministic channel model to the Gaussian channel model, and also derive two upper bounds on the constrained sum-capacity. Numerical evaluations show that the constrained sum-capacity in the Gaussian channels oscillates between the sum-capacity with Gaussian Han-Kobayashi schemes and that with single message schemes.**

I. Introduction

We consider the sum-rate maximization problem in two-user Gaussian interference channels under the constraints of successive decoding. While the information theoretic capacity region of the Gaussian interference channel is still not known, it has been shown that a Han-Kobayashi scheme with random Gaussian codewords can achieve within 1 bit/s/Hz of the capacity region [4]. In this scheme, each user decodes both users' common messages jointly, and then decodes its own private message. In comparison, the simplest decoding constraint is treating the interference from the other users as noise. It has been shown that within a certain range of channel parameters for *weak* interference channels, treating interference as noise achieves the information theoretic sum-capacity [1], [6], [7].

In this paper, we consider a decoding constraint — *successive decoding of Gaussian superposition codewords* — that bridges the complexity between joint decoding (e.g. in Han-Kobayashi schemes) and treating interference as noise. We investigate the constrained sum-capacity and how to achieve it. To clarify and capture the key aspects of the problem, we resort to a deterministic channel model [2]. In [3], the information theoretic capacity region for the two-user deterministic interference channel is derived as a special case of the El Gamal-Costa deterministic model [5], and is shown to be achievable using Han-Kobayashi schemes.

To capture the use of successive decoding of Gaussian codewords, we introduce the *complementarity conditions* on the bit levels in the deterministic formulation. We develop transmission schemes on the bit-levels, which in the Gaussian model corresponds to message splitting and power allocation of the messages. We then solve the constrained sum-capacity, and show that it *oscillates* (as a function of the cross link gain parameters) between the information theoretic sum-capacity and the sum-capacity with interference treated as noise. Furthermore, the minimum number of messages needed to achieve the constrained sum-capacity is obtained. We show that if the number of messages is limited to even *one less* than this minimum capacity achieving number, the sum-capacity drops to that with interference treated as noise.

We then translate the optimal scheme in the deterministic interference channel to the Gaussian channel, using a rate constraint equalization technique. To evaluate the optimality of the translated achievable schemes, we derive two upper bounds on the sum-capacity with Gaussian Han-Kobayashi schemes, which automatically apply to the sum-capacity with successive decoding schemes. The two bounds are shown to be tight in different ranges of parameters.

The remainder of the paper is organized as follows. Section II formulates the problem of sum-capacity with successive decoding of Gaussian codewords in Gaussian interference channels. Section III reformulates the problem with the deterministic channel model, and then solves the constrained sum-capacity. Section IV translates the optimal schemes in the deterministic channel back to the Gaussian channel, and derives two upper bounds on the constrained sum-capacity. Conclusions are drawn in Section V. Due to space limitations, all the proofs are omitted here, and can be found in [9].

II. Problem formulation in Gaussian channels

We consider the two-user Gaussian interference channel:

$$y_1 = h_{11}x_1 + h_{21}x_2 + z_1,$$
$$y_2 = h_{22}x_2 + h_{12}x_1 + z_2,$$

where $\{h_{ij}\}$ are constant complex channel gains, and $z_i \sim \mathcal{CN}(0, N_i)$. Define $g_{ij} \triangleq |h_{ij}|^2, (i, j = 1, 2)$. There is an average power constraint equal to $\bar{p}_i$ for the i^{th} user $(i = 1, 2)$. Suppose the i^{th} user uses a superposition of L_i codewords $x_i^{(\ell)} (1 \leq \ell \leq L_i)$ to generate the transmit signal x_i,

$$x_i = \sum_{\ell=1}^{L_i} \sqrt{p_i^{(\ell)}} x_i^{(\ell)},$$

$$\sum_{\ell=1}^{L_i} p_i^{(\ell)} \le \bar{p}_i, \quad i = 1, 2, \tag{1}$$

where each $x_i^{(\ell)}$ has a block length n, and is chosen from a codebook generated by using IID random variables of $\mathcal{CN}(0, 1)$. The i^{th} receiver attempts to decode all $x_i^{(\ell)}$, $\ell = 1, \ldots, L_i$, using successive decoding. Denote by $\mathcal{O}_i$ its decoding order of the $L_1 + L_2$ messages from both users. Denote the message that has order q in $\mathcal{O}_i$ by $x_{t_{q,i}}^{(\ell_{q,i})}$, i.e., it is the $\ell_{q,i}{}^{th}$ message of the $t_{q,i}{}^{th}$ user. Denote by $r_i^{(\ell)}$ the rate of message $x_i^{(\ell)}$. Then, treating undecoded messages as noise, for the successive decoding procedure to have a vanishing error probability as $n \to \infty$, we have the following constraints:

$$r_{t_{q,i}}^{(\ell_{q,i})} \le \log\left(1 + \frac{p_{t_{q,i}}^{(\ell_{q,i})} g_{t_{q,i}i}}{\sum_{s=q+1}^{L_1+L_2} p_{t_{s,i}}^{(\ell_{s,i})} g_{t_{s,i}i} + N_i}\right), \tag{2}$$

$$\forall 1 \le q \le \max_{1 \le \ell \le L_i} \{\text{order of } x_i^{(\ell)} \text{ in } \mathcal{O}_i\}, \ i = 1, 2.$$

Now, we formulate the sum-rate maximization problem as:

$$\max_{\substack{\{p_i^{(\ell)}\}, \mathcal{O}_i, \\ i=1,2}} \sum_{i=1}^{2} \sum_{\ell=1}^{L_i} r_i^{(\ell)} \tag{3}$$

subject to: $(1), (2)$.

Note that problem (3) involves both a *combinatorial optimization* of the decoding orders $\{\mathcal{O}_i\}$ and a *non-convex optimization* of the transmit power $\{p_i^{(\ell)}\}$. Thus, it is a hard problem from an optimization point of view, which has not been addressed in the literature. Interestingly, we show that an "indirect" approach can effectively and fruitfully provide approximately optimal solutions to the above problem (3). Instead of directly working with the Gaussian model, we approximate the problem using the recently developed deterministic channel model [2]. We give a complete analytical solution that achieves the constrained sum-capacity in all channel parameters. Then, we translate it back to the Gaussian formulation (3) to get approximately optimal solutions.

III. Sum-capacity in deterministic interference channels

A. Channel Model and Problem Formulation

In this section, we apply the deterministic channel model [2] as an approximation of the Gaussian model on the two-user interference channel. Fig. 1 depicts the desired signal and the interference signal at the two receivers. For $i = 1, 2$ and $j \ne i$, we define

$$n_{ii} \triangleq \log(\mathsf{SNR}_i) = \log(\frac{g_{ii}\bar{p}_i}{N_i}),$$

$$n_{ij} \triangleq \log(\mathsf{INR}_i) = \log(\frac{g_{ji}\bar{p}_j}{N_i}),$$

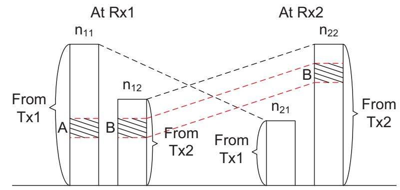

Fig. 1. Two-user deterministic interference channel. Levels A and B interfere at the 1^{st} receiver, and cannot be fully active simultaneously.

where n_{ij} denotes the number of bit levels of the signal sent from the j^{th} transmitter that are above the noise level at the i^{th} receiver. WLOG, we assume that $n_{11} \ge n_{22}$. Further, we define

$$\delta_1 \triangleq n_{11} - n_{21}, \ \delta_2 \triangleq n_{22} - n_{12}, \tag{4}$$

which represent the cross channel gains relative to the direct channel gains, in terms of the number of bit-level shifts.

In the original formulation of the deterministic channel model [2], $\{n_{ij}\}$ are *integers*, and the achievable scheme must also have integer bit-levels. Here, in formulating the optimization problem, we consider $\{n_{ij}\}$ to be *real*, which denote the amount of *information levels*. We will show that this relaxation gives integer bit-level optimal solutions whenever $\{n_{ij}\}$ are integers (cf. Remark 2 later). A concise representation of this formulation is provided in Figure 2:

- The sets of information levels of the *desired* signals at receivers 1, 2 are represented by the intervals $I_1 = [0, n_{11}]$ and $I_2 = [n_{11} - n_{22}, n_{11}]$ on two parallel lines, where the leftmost points correspond to the most significant levels, and the points at n_{11} correspond to the positions of the noise levels at both receivers.
- The positions of the information levels of the *interfering* signals are indicated by the dashed lines crossing between the two parallel lines.

An information level is a real *point*, and the measure of a set of levels equals the amount of information that this set can carry. The design variables in the deterministic channel are *whether each level of a user carries information or not*, characterized by the following indicator function definition $f_i(x)$ $(i = 1, 2)$:

Definition 1:

$$f_i(x) = \begin{cases} 1, & \text{if } x \in I_i, \text{ and level } x \text{ carries} \\ & \text{information for the } i^{th} \text{ user,} \\ 0, & \text{otherwise.} \end{cases} \tag{5}$$

With this model, the rates of the two users are

$$R_1 = \int_0^{n_{11}} f_1(x)\mathrm{d}x, \ R_2 = \int_0^{n_{11}} f_2(x)\mathrm{d}x.$$

For an information level x s.t. $f_i(x) = 1$, we call it an *active* level for the i^{th} user, and otherwise an *inactive* level.

The constraints from successive decoding of Gaussian codewords translate to the following *Complementarity Conditions* in the deterministic formulation:

$$f_1(x)f_2(x + \delta_1) = 0, \forall -\infty < x < \infty, \tag{6}$$

$$f_2(x)f_1(x + \delta_2) = 0, \forall -\infty < x < \infty. \tag{7}$$

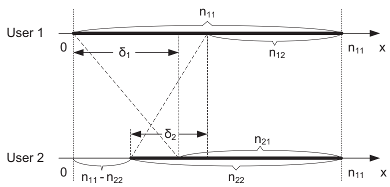

Fig. 2. Interval representation of the two-user deterministic interference channel.

The interpretation of (6) and (7) are as follows: for any two levels each from one of the two users, if they interfere with each other at *any* of the two receivers, they cannot be simultaneously active. For example, in Fig. 1, information levels A from the 1^{st} user and B from the 2^{nd} user interfere at the 1^{st} receiver, and hence cannot be fully active simultaneously. These complementarity conditions have also been characterized using a conflict graph model in [8].

Remark 1: Given any $f_i(x), x \in I_i$, every disjoint segment within I_i with $f_i(x) = 1$ on it corresponds to a distinct message. Adjacent segments that can be so combined as a super-segment having $f_i(x) = 1$ on it, are viewed as *one* segment, i.e., the combined super-segment.

Finally, we note that

$$\begin{aligned} &(6) \Leftrightarrow f_2(x)f_1(x-\delta_1) = 0, \forall -\infty < x < \infty, \\ \text{and} \quad &(7) \Leftrightarrow f_1(x)f_2(x-\delta_2) = 0, \forall -\infty < x < \infty. \end{aligned}$$

Thus, we have the following result:

Lemma 1: $\left\{ \begin{array}{l} \delta_1 = a \\ \delta_2 = b \end{array} \right.$ and $\left\{ \begin{array}{l} \delta_1 = -b \\ \delta_2 = -a \end{array} \right.$ correspond to the same set of complementarity conditions.

We consider the problem of maximizing the sum-rate of the two users employing successive decoding, formulated as the following infinite dimensional optimization problem:

$$\max_{f_1(x), f_2(x)} \int_0^{n_{11}} f_1(x) + f_2(x) \mathrm{d}x \qquad (8)$$
$$\text{subject to } (5), (6), (7).$$

B. Symmetric Interference Channels

In this section, we consider the case where $n_{11} = n_{22}, n_{12} = n_{21}$. Define $\alpha \triangleq \frac{n_{12}}{n_{11}}, \beta \triangleq 1 - \alpha = \delta_1 = \delta_2$. WLOG, we normalize the amount of information levels by n_{11}, and consider $n_{11} = n_{22} = 1$, and $n_{12} = n_{21} = \alpha$.

From Lemma 1, *it is sufficient to only consider the case with* $\beta \geq 0$*, i.e.* $\alpha \leq 1$. We next derive the constrained sum-capacity using successive decoding for $\alpha \in [0, 1]$, first without upper bounds on the number of messages, then with upper bounds. As the constrained sum-capacity is achievable with $R_1 = R_2$, we also use the maximum achievable *symmetric* rate, denoted by $R(\alpha)$ as a function of α, as an equivalent performance measure. $R(\alpha)$ is thus one half of the optimal value of (8).

1) Symmetric Capacity without Constraint on the Number of Messages:

Theorem 1: The maximum achievable symmetric rate using successive decoding, (i.e., having constraints (6), (7)), $R(\alpha)$ $(\alpha \in [0, 1])$, is characterized by

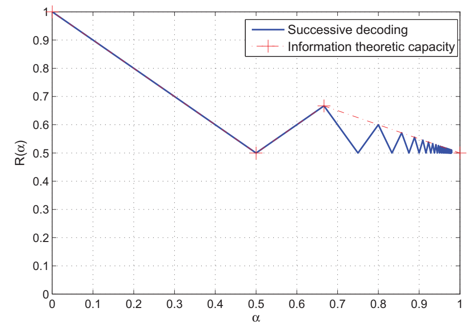

Fig. 3. The symmetric capacity with successive decoding in symmetric deterministic interference channels.

- $R(\alpha) = 1 - \frac{\alpha}{2}$, when $\alpha = \frac{2n}{2n+1}, n = 0, 1, 2, \ldots$.
- $R(\alpha) = \frac{1}{2}$, when $\alpha = \frac{2n-1}{2n}, n = 1, 2, 3, \ldots$.
- In every interval $[\frac{2n}{2n+1}, \frac{2n+1}{2n+2}], n = 0, 1, 2, \ldots$, $R(\alpha)$ is a decreasing linear function.
- In every interval $[\frac{2n-1}{2n}, \frac{2n}{2n+1}], n = 1, 2, 3, \ldots$, $R(\alpha)$ is an increasing linear function.
- $R(1) = \frac{1}{2}$.

$R(\alpha)$ is plotted in Fig. 3, compared with the information theoretic capacity [3]. We divide the interval $[0, 1]$ into consecutive segments $s_1, s_2, \ldots$ such that $|s_1| = |s_2| = \ldots = \beta$, with the last segment ending at 1 having the length of the proper residual (cf. Fig. 4). Define

$$\mathcal{G}_1 \triangleq \bigcup_{i=1,2,\ldots} s_{2i-1} \text{ and } \mathcal{G}_2 \triangleq \bigcup_{i=1,2,\ldots} s_{2i}. \qquad (9)$$

We then have the following optimal scheme that achieves the constrained symmetric-capacity $R(\alpha)$:

Corollary 1: When $\alpha \in (0, 1)$, the constrained symmetric capacity is achievable with

$$f_1(x) = f_2(x) = \left\{ \begin{array}{ll} 1, & \forall x \in \mathcal{G}_1 \\ 0, & \forall x \in \mathcal{G}_2 \end{array} \right. . \qquad (10)$$

In the special cases when $\alpha = \frac{2n-1}{2n}, (n = 1, 2, \ldots,)$ and $\alpha = 1$, the constrained symmetric-capacity drops to $\frac{1}{2}$, achievable by simply time sharing $\left\{ \begin{array}{ll} f_1(x) = 1, & x \in [0, 1] \\ f_2(x) = 0, & x \in [0, 1] \end{array} \right.$ and $\left\{ \begin{array}{ll} f_1(x) = 0, & x \in [0, 1] \\ f_2(x) = 1, & x \in [0, 1] \end{array} \right.$.

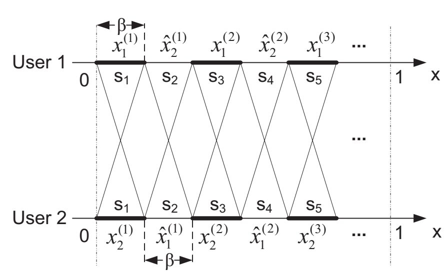

Fig. 4. The optimal scheme in the symmetric deterministic interference channel.

We observe that the *numbers of messages used* by the two users – L_1, L_2 – in the optimal scheme (10) are as follows:

Corollary 2:

- when $\alpha \in (\frac{2n-1}{2n}, \frac{2n+1}{2n+2}), (n = 1, 2, \ldots)$, $L_1 = L_2 = n+1$;
- when $\alpha \in [0, \frac{1}{2}]$, $\alpha = \frac{2n-1}{2n}, (n = 1, 2, \ldots)$, or $\alpha = 1$, $L_1 = L_2 = 1$.

Remark 2: In the case where $\{n_{ij}\}$ are *integers*, $\alpha = \frac{n_{12}}{n_{11}}$ is a *rational* number. As a result, the optimal scheme (10) consists of active segments $\mathcal{G}_1$ that have rational boundaries with the same denominator n_{11}. This corresponds to an *integer* bit-level solution.

From Theorem 1 (cf. Fig. 3), it is interesting to see that the symmetric capacity oscillates as a function of α between the information theoretic capacity and the baseline of $1/2$. This phenomenon is a consequence of the complementarity conditions (6), (7).

2) The Case with a Limited Number of Messages:

Now, we consider the case when there are constraints on the maximum number of messages for the two users. We start with the following two lemmas:

Lemma 2: If there exists a segment with an even index s_{2i} $(i \geq 1)$ and s_{2i} does not end at 1, such that $f_1(x) = 1, \forall x \in s_{2i},$ *or* $f_2(x) = 1, \forall x \in s_{2i}$, then $R_1 + R_2 \leq 1$.

Lemma 3: If there exists a segment with an odd index s_{2i-1} $(i \geq 1)$, such that $f_1(x) = 0, \forall x \in s_{2i-1},$ *or* $f_2(x) = 0, \forall x \in s_{2i-1}$, then $R_1 + R_2 \leq 1$.

Recall that the optimal scheme (10) requires that, for both users, *all* segments in $\mathcal{G}_2$ are fully inactive, and *all* segments in $\mathcal{G}_1$ are fully active. The above two lemmas show the cost of violating (10): if one of the segments in $\mathcal{G}_2$ becomes fully active for either user (cf. Lemma 2), or one of the segments in $\mathcal{G}_1$ becomes fully inactive for either user (cf. Lemma 3), the resulting sum-rate cannot be greater than 1.

Lemmas 2 and 3 lead to the following theorem:

Theorem 2: Denote by $L_i (i = 1, 2)$ the number of messages used by the i^{th} user. When $\alpha \in (\frac{2n-1}{2n}, \frac{2n+1}{2n+2}), (n = 1, 2, \ldots,)$ if $L_1 \leq n$ *or* $L_2 \leq n$, the maximum achievable sum-rate is 1.

Comparing Theorem 2 with Corollary 2, we conclude that if the number of messages used for *either* user is fewer than the number used in the optimal scheme as in Corollary 2, the maximum achievable symmetric rate drops to $\frac{1}{2}$. An illustration with $L_1 \leq 2$ (or $L_2 \leq 2$) is plotted in Fig. 5.

Complete solutions in *asymmetric* channels follow similar ideas. The details can be found in [9].

IV. Approximate Sum-capacity in Gaussian Interference Channels

In this section, we turn our focus back to the two-user Gaussian interference channel, and consider the sum-rate maximization problem (3). We *translate* the optimal solution of the deterministic channel into the Gaussian channel, and derive upper bounds on the optimal value of (3). We then evaluate the achievability of our translation against these upper bounds.

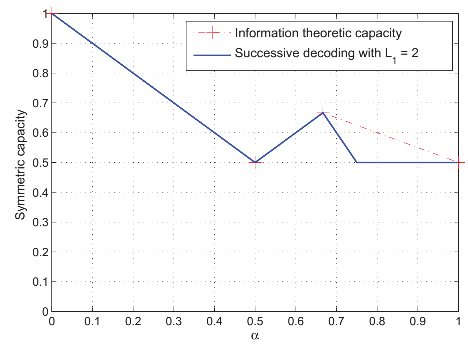

Fig. 5. Maximum achievable symmetric rate with $L_1 \leq 2$.

A. Achievable Sum-rate Motivated by the Optimal Scheme in the Deterministic Channel

Consider symmetric interference channels: $g_{11} = g_{22}, g_{12} = g_{21}, N_1 = N_2, \bar{p}_1 = \bar{p}_2 = \bar{p}$. WLOG, we assume that $N_1 = 1$ and $g_{11} = 1$. We consider the case where the cross channel gain is no greater than the direct channel gain: $0 \leq g_{12} \leq g_{11}$. We note that the achievable schemes for general asymmetric channels can be derived similarly, albeit more tediously.

In the optimal deterministic scheme, the key property that ensures optimality is the following (cf. Fig. 4):

Corollary 3: For any common message $x_i^{(\ell)}$, it is subject to the same achievable rate constraint at both receivers.

For example, message $x_1^{(1)}$ is subject to an achievable rate constraint of $|x_1^{(1)}|$ at the 1^{st} receiver, and that of $|\hat{x}_1^{(1)}|$ at the 2^{nd} receiver, with $|x_1^{(1)}| = |\hat{x}_1^{(1)}| = \beta$. Motivated by Corollary 3, we translate the optimal deterministic scheme to the power allocation of the messages by *equalizing the two rate constraints* for every common message. We propose the following power allocation algorithm that equalizes the rate constraints, in which L counts the number of messages used by each user. The derivations can be found in [9].

Algorithm 1

Initialize $L = 1$.
  Step 1: If $\bar{p} \leq \frac{1-g_{12}}{g_{12}^2}$, then $p^{(L)} \leftarrow \bar{p}$ and terminate.
  Step 2: $p^{(L)} \leftarrow 1 - g_{12} + (1 - g_{12}^2)\bar{p}$. $L \leftarrow L + 1$.
    $\bar{p} \leftarrow \bar{p} - p^{(1)}$. Go to Step 1.

The decoding orders at both receivers then follow the same ones as in the deterministic channel (cf. Fig. 4).

B. Upper Bounds on the Sum-capacity with Successive Decoding of Gaussian Codewords

In this subsection, we provide two upper bounds on the optimal value of (3) for general (asymmetric) channels. More specifically, the bounds are derived for the sum-capacity with Gaussian Han-Kobayashi schemes, which automatically upper bound the sum-capacity with successive decoding of Gaussian codewords, (as Gaussian superposition coding - successive decoding is a special case of Han-Kobayashi schemes [9]). The bounds are obtained by selecting two subsets of the

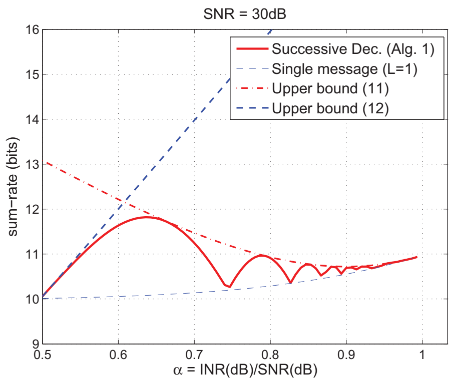

Fig. 6. Performance evaluation: achievability vs. upper bounds.

inequality constraints that characterize the Han-Kobayashi capacity region. Maximizing the sum-rate with each of the two subsets of inequalities leads to one of the two upper bounds.

For the i^{th} user ($i = 1, 2$), we denote by q_i and $\bar{p}_i - q_i$ the power allocated to its private and common messages. WLOG, we normalize the channel parameters such that $g_{11} = g_{22} = 1$.

Theorem 3: The sum-capacity with Gaussian Han-Kobayashi schemes is upper bounded by

$$\max_{q_1,q_2} \min \left\{ \log\left(1 + \frac{\bar{p}_1 + g_{21}(\bar{p}_2 - q_2)}{g_{21}q_2 + N_1}\right) + \log\left(1 + \frac{q_2}{g_{12}q_1 + N_2}\right), \log\left(1 + \frac{\bar{p}_2 + g_{12}(\bar{p}_1 - q_1)}{g_{12}q_1 + N_2}\right) + \log\left(1 + \frac{q_1}{g_{21}q_2 + N_1}\right) \right\}, \quad (11)$$

and

$$\max_{q_1,q_2} \log\left(1 + \frac{q_1 + g_{21}(\bar{p}_2 - q_2)}{g_{21}q_2 + N_1}\right) + \log\left(1 + \frac{q_2 + g_{12}(\bar{p}_1 - q_1)}{g_{12}q_1 + N_2}\right). \quad (12)$$

C. Performance Evaluation

We numerically evaluate our results in a symmetric Gaussian interference channel with an SNR of $30dB$. To evaluate the performance of successive decoding, we sweep the parameter range of $\alpha = \frac{\log(\mathsf{INR})}{\log(\mathsf{SNR})} \in [0.5, 1]$, as when $\alpha \in [0, 0.5]$, an approximate optimal transmission scheme is simply treating interference as noise without successive decoding.

In Fig. 6, the achievable sum-rate for Algorithm 1 and the two upper bounds (11), (12) are evaluated. The maximum achievable sum-rate with a *single* message for each user is also computed, and is used as a baseline scheme for comparison.

We make the following observations:

- The first upper bound (11) is tighter for higher INR while the second upper bound (12) is tighter for lower INR.
- The constrained sum-capacity with successive decoding of Gaussian codewords oscillates between the sum-capacity with Han-Kobayashi schemes and that with single message schemes.
- The largest difference between the sum-capacity of successive decoding and that of single message schemes is about 1.8 bits, appearing at around $\frac{\log(\mathsf{INR})}{\log(\mathsf{SNR})} = 0.64$.
- The largest difference between the sum-capacity of successive decoding and that of Han-Kobayashi schemes is about 1.0 bits, appearing at around $\frac{\log(\mathsf{INR})}{\log(\mathsf{SNR})} = 0.75$.

It is worth noting that although the above differences (1.8 bits and 1.0 bits) with SNR $= 30dB$ may not seem very significant, as SNR $\to \infty$, both differences will go to infinity [9].

V. Conclusions

In this paper, we studied the problem of sum-rate maximization with Gaussian superposition coding and successive decoding in two-user interference channels. We used the deterministic channel model as an educated approximation of the Gaussian channel model, and introduced the complementarity conditions that capture the use of successive decoding of Gaussian codewords. We solved the constrained sum-capacity in the deterministic interference channel, and obtained the capacity achieving schemes with the minimum number of messages. The constrained sum-capacity oscillates as a function of the cross link gain parameters between the information theoretic sum-capacity and the sum-capacity with interference treated as noise. Furthermore, we showed that if the number of messages used by either of the two users is fewer than its minimum capacity achieving number, the maximum achievable sum-rate drops to that with interference treated as noise. We translated the optimal schemes in the deterministic channel to the Gaussian channel using a rate constraint equalization technique, and provided two upper bounds on the constrained sum-capacity with successive decoding of Gaussian codewords. Numerical evaluations of the translation and the upper bounds showed that the constrained sum-capacity oscillates between the sum-capacity with Han-Kobayashi schemes and that with single message schemes.

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