# Linear Diversity-Embedding STBC : Design Issues and Applications

K.M.Z. Islam*, P. Rabiei*, N. Al-Dhahir*, S.N. Diggavi†, A.R. Calderbank‡

**Abstract**

We design a novel class of space-time codes, called linear diversity-embedding space-time block codes (LDE-STBC) where a high-rate STBC is linearly superimposed on a high-diversity STBC without requiring channel knowledge at the transmitter. In applying this scheme to multimedia wireless communications, each traffic type constitutes a transmission layer that operates at a suitable rate-diversity tradeoff point according to its quality-of-service requirements. This, in turn, provides an unequal-error-protection (UEP) capability to the different information traffic types and allows a form of wireless communications where the high-rate STBC opportunistically takes advantage of good channel realizations while the embedded high-diversity STBC ensures that at least part of the information is decoded reliably.

We investigate transceiver design issues specific to LDE-STBC including reduced-complexity coherent decoding and effective schemes to vary the coding gain to further enhance UEP capabilities of the code. Furthermore, we investigate the application of LDE-STBC to wireless multicasting and demonstrate its performance advantage over conventional equal-error-protection STBC.

## I. Introduction

There is a fundamental tradeoff between rate and reliability (diversity) in multiple-antenna wireless communications both in multiplexing-rate [27] as well as in fixed-rate codes [23]. In [10], [12], we demonstrated that diversity is a systems resource that can be allocated judiciously to tradeoff rate against reliability in multimedia wireless communications. Multimedia data is typically made up of components with different rate/reliability requirements. A real-time data stream may require more diversity (protection) than non-real-time data. A complex data stream such as compressed images may involve different components (e.g. coarse/fine details) that require

[1]N. Al-Dhahir is the contact author. Email : aldhahir@utdallas.edu. This work was suppported in part by NSF contracts CCF 0430654 and DMS 0528010.

* University of Texas at Dallas, USA †EPFL, Switzerland ‡Princeton University, USA

different levels of error protection. Over-provisioning of diversity to one data component will mean the loss of rate to that and other data components; a waste of system resources. We introduced in [10], [12] a new class of space-time codes that have a high-diversity layer embedded within a high-rate layer. This represents a paradigm shift from conventional space-time codes [23], [24] designed to provide the same maximum diversity order to all information symbols irrespective of their individual quality-of-service (QoS) requirements. Even when all information symbols have the same QoS requirements, our diversity-embedding codes can provide a throughput advantage over conventional space-time codes for the following reason. The high-rate layer **opportunistically** takes advantage of good channel realizations while the embedded high-diversity layer ensures that its information symbols are decoded reliably. Hence, our codes achieve a high throughput by riding over the channel peaks without requiring channel knowledge at the transmitter. Using our proposed coding scheme, information symbols are transmitted over space and time in layers using multiple transmit antennas where each layer is designed to operate at a prescribed rate and diversity level. Hence, our codes enjoy a UEP capability.

UEP schemes have a rich history and have been extensively investigated for multimedia communications. Since the early work in [19] on linear UEP coding, several UEP schemes have been proposed including those based on rate-compatible punctured convolutional codes [14], [6], multi-level trellis codes [3], and more recently, low-density parity-check codes (LDPC) [21]. Other UEP schemes include hierarchical modulation (see [5], [25], [3] and references therein) which is adopted in digital video broadcasting (DVB) standards. Recently, UEP scheme for multiple-antenna systems were investigated in [22], [17], [16]. UEP is achieved in [22] by switching between different STBC designs and using punctured turbo codes. The UEP scheme in [17] is also based on time-sharing between STBC's with different diversity levels. However, for each STBC, the *same* diversity level is provided to all information symbols. The UEP scheme in [16] is based on differential transmission of unitary STBC codewords and the use of non-uniformly-spaced PSK constellations. Unlike these schemes, diversity-embedded coding, introduced in [10], [12] does not use time-sharing of different STBC's and provides UEP even with standard uniformly-spaced signal constellations. Furthermore, DE-STBC introduces spatio-temporal correlations in a carefully-designed manner to provide multiple diversity levels to the information symbols within the *same* codeword.

This work complements our work in [12] in that the focus in [12] was non-linear diversity-embedding codes based on set-partitioning principles while the focus here is linear diversity-embedding space-time block codes (LDE-STBC) based on the superposition principle due to their reduced decoding complexity. Furthermore, all of the linear code constructions in [10], [12] were for $4$ transmit antennas and $2$ diversity layers while in this work we present new constructions for 2 and 3 transmit antennas and for 3 diversity layers. The main contributions of this paper are summarized as follows. We present in Section II a new class of constellation-dependent LDE-STBC constructions where the information layers are transmitted at different power levels (while still satisfying the overall transmit power constraint) to meet the prescribed rate-diversity levels of the individual layers while achieving a desirable tradeoff between their coding gains. The relative power levels are optimized offline (as a function of the signal constellation size) taking into account the peak-to-minimum power ratio of the transmitted signal. Furthermore, this novel power scaling scheme allows us to develop new LDE-STBC constructions for 2 and 3 transmit antennas and for 3 diversity layers. We show in Section III how to reduce the decoding complexity of LDE-STBC significantly by exploiting its algebraic structure. Finally, we investigate the multicasting application where LDE-STBC demonstrates appreciable performance advantages over conventional single-layer STBC.

## II. Diversity-Embedding Space-Time Block Coding

### A. Transmission Model

We consider the transmission scenario where information symbols are space-time block-encoded over $M_t$ transmit antennas, transmitted through quasi-static Rayleigh flat-fading channels, and received by $M_r$ antennas. Furthermore, we assume that the transmitter has no channel state information (CSI) while the receiver performs coherent decoding (with perfect or estimated CSI). The DE-STBC is designed over $T$ transmission symbols and the received signal after demodulation and sampling can be written as

$$\mathbf{Y} = \mathbf{H}\mathbf{X} + \mathbf{Z} \tag{1}$$

where $\mathbf{Y} = [\mathbf{y}(0) \dots \mathbf{y}(T-1)] \in \mathbb{C}^{M_r \times T}$ is the space-time received signal matrix, $\mathbf{H} \in \mathbb{C}^{M_r \times M_t}$ is the quasi-static [1] Rayleigh flat-fading space-time channel matrix with independent identically distributed entries, $\mathbf{X} = [\mathbf{x}(0) \dots \mathbf{x}(T-1)] \in \mathbb{C}^{M_t \times T}$ is the transmitted STBC with transmit power constraint $P = \frac{E[tr(\mathbf{X}\mathbf{X}^*)]}{M_t}$, and $\mathbf{Z} = [\mathbf{z}(0) \dots \mathbf{z}(T-1)] \in \mathbb{C}^{M_r \times T}$ consists of additive white (temporally and spatially) Gaussian noise samples with variance $\sigma^2$. A transmission scheme with diversity order $d$ has an error probability at high SNR behaving as $\bar{P}_e(\text{SNR}) \approx \text{SNR}^{-d}$; i.e., diversity order represents the *slope* (at high SNR) of the error rate versus SNR curve on a log-log scale.

[1] The channel is assumed fixed over a coherence interval of $T$ symbols and varies independently from one coherence interval to the next one.

### B. Rate-Diversity Tradeoff for Fixed Signal Constellations

For a Rayleigh flat-fading channel, the following theorem illustrates the rate-diversity tradeoff (tension) for a fixed signal constellation [23], [18]. We emphasize that this tradeoff relationship is different from the rate-diversity tradeoff in [27] where the transmit constellation size grows with SNR. Our focus in this paper is exclusively on the scenario where the signal constellation size is the same for all SNR.

*Theorem 2.1:* If the achievable spatial diversity order is $qM_r$ with a signal constellation $\mathcal{S}$ of size $M = |\mathcal{S}|$, then the achievable rate $R$ in bits per channel use (PCU) is upper-bounded as follows

$$R \leq (M_t - q + 1) \log_2 |\mathcal{S}| \ . \tag{2}$$

### C. DE-STBC Design Criteria

We consider the space-time transmission scenario where information symbols are selected from standard signal constellations (e.g. QAM or PSK) and transmitted in *layers* each characterized by a prescribed rate-diversity operating point according to its QoS requirements. To simplify the discussion, we focus on the case of 2 layers. Let $\mathcal{A}$ denote the message set for the first (higher-diversity) information layer and $\mathcal{B}$ denote that for the second (lower-diversity) information layer. We denote the achievable rates for the two diversity layers by $R(\mathcal{A})$ and $R(\mathcal{B})$, respectively. We assume (unless otherwise stated) a maximum likelihood (ML) decoder that jointly decodes the two diversity layers with average error probabilities, $\bar{P}_e(\mathcal{A})$ and $\bar{P}_e(\mathcal{B})$, respectively.

*Definition 2.2:* We design the DE-STBC $\mathbf{X}_{\mathbf{a},\mathbf{b}}$ such that a certain rate-diversity tuple $(R_a, D_a, R_b, D_b)$ is achievable, where $R_a = R(\mathcal{A}) = \frac{\log_2(|\mathcal{A}|)}{T}$, $R_b = R(\mathcal{B}) = \frac{\log_2(|\mathcal{B}|)}{T}$ and $\mathbf{a}$, $\mathbf{b}$ are message vectors composed of the information symbols transmitted from Layers $\mathcal{A}$ and $\mathcal{B}$, respectively. Analogous to [27] we define

$$D_a = \lim_{SNR \to \infty} \frac{-\log \bar{P}_e(\mathcal{A})}{\log(SNR)}, \ D_b = \lim_{SNR \to \infty} \frac{-\log \bar{P}_e(\mathcal{B})}{\log(SNR)}. \tag{3}$$

It was shown in [10], [12] that to guarantee the diversity orders $D_a, D_b$ we must have

*Theorem 2.3:*

$$\min_{\mathbf{a}_1 \neq \mathbf{a}_2 \in \mathcal{A}} \ \min_{\mathbf{b}_1, \mathbf{b}_2 \in \mathcal{B}} \mathrm{r}ank(\mathbf{G}(\mathbf{X}_{\mathbf{a}_1,\mathbf{b}_1}, \mathbf{X}_{\mathbf{a}_2,\mathbf{b}_2})) = D_a/M_r \tag{4}$$

$$\min_{\mathbf{b}_1 \neq \mathbf{b}_2 \in \mathcal{B}} \ \min_{\mathbf{a}_1, \mathbf{a}_2 \in \mathcal{A}} \mathrm{r}ank(\mathbf{G}(\mathbf{X}_{\mathbf{a}_1,\mathbf{b}_1}, \mathbf{X}_{\mathbf{a}_2,\mathbf{b}_2})) = D_b/M_r \tag{5}$$

where $\mathbf{G}(\mathbf{X}_{\mathbf{a}_1,\mathbf{b}_1}, \mathbf{X}_{\mathbf{a}_2,\mathbf{b}_2}) = \mathbf{X}_{\mathbf{a}_1,\mathbf{b}_1} - \mathbf{X}_{\mathbf{a}_2,\mathbf{b}_2}$ is the codeword difference matrix. In words, if we transmit a particular message from Layer $\mathcal{A}$, regardless of which message is transmitted from Layer $\mathcal{B}$, a diversity level of $D_a$ is guaranteed for all symbols in Layer $\mathcal{A}$. A similar argument holds for Layer $\mathcal{B}$ and this argument extends to more than 2 layers in a straightforward manner. It follows from the definition of matrix rank that

$\max(D_a, D_b) \leq M_r \min(M_t, T)$. The criteria in (4) and (5) can be viewed as generalizations of the rank criterion in [23] which follows as a special case by setting $D_a = D_b = M_t M_r$; i.e. the single-layer STBC is designed in [23], [24] to ensure equal error protection by providing maximum diversity order for *all* information symbols at the expense of rate (e.g. as in Orthogonal Designs [24]).

*Definition 2.4:* The coding gains (CG) achievable by diversity layers $\mathcal{A}$ and $\mathcal{B}$ are defined as follows

$$CG_{\mathcal{A}} = \min_{\mathbf{a}_1 \neq \mathbf{a}_2 \in \mathcal{A}} \min_{\mathbf{b}_1, \mathbf{b}_2 \in \mathcal{B}} \prod_{i=1}^{D_a} \lambda_i \tag{6}$$

$$CG_{\mathcal{B}} = \min_{\mathbf{b}_1 \neq \mathbf{b}_2 \in \mathcal{B}} \min_{\mathbf{a}_1, \mathbf{a}_2 \in \mathcal{A}} \prod_{i=1}^{D_b} \lambda_i \tag{7}$$

where $\lambda_i$ denote the non-zero eigenvalues of the Hermitian positive semi-definite matrix $\mathbf{G}\mathbf{G}^*$, where we suppress the argument of $\mathbf{G}$ henceforth, compared to (4) and (5), to simplify notation.

### D. Linear DE-STBC Constructions

In this subsection, we present several DE-STBC constructions for 2,3, and 4 transmit antennas and for 2 and 3 diversity layers. Our focus in this paper is on *linear* (over the complex field) additive constructions where the transmitted codewords are given by the *sum* of the codewords from each diversity layer (i.e. $\mathbf{X}_{\mathbf{a},\mathbf{b}} = \mathbf{X}_{\mathbf{a}} + \mathbf{X}_{\mathbf{b}}$) whose entries are drawn from standard signal constellations (e.g. PSK or QAM). The linearity constraint is imposed to reduce decoding complexity (see Section III). Non-linear constructions were investigated in [12].

*1) Constellation-Independent Constructions:* In this first class of LDE-STBC constructions, the achievable rate-diversity tuple is *independent* of the signal constellation.

**Example 1:** Here Layer $\mathcal{A}$ contains 3 information symbols $\{a(0), a(1), a(2)\} \in \mathcal{S}$ and Layer $\mathcal{B}$ contains only 1 information symbol $b(0) \in \mathcal{S}$. Hence, $\mathbf{a} = \begin{bmatrix} a(0) & a(1) & a(2) \end{bmatrix}$, $\mathbf{b} = [b(0)]$, $|\mathcal{A}| = |\mathcal{S}|^3$,$|\mathcal{B}| = |\mathcal{S}|$. This example achieves the tuple $(\frac{3}{4}\log_2|\mathcal{S}|, 4M_r, \frac{1}{4}\log_2|\mathcal{S}|, M_r)$.

$$\mathbf{X}_{\mathbf{a},\mathbf{b}} = \mathbf{X}_{\mathbf{a}} + \mathbf{X}_{\mathbf{b}} = \begin{bmatrix} a(0) & a(1) & a(2) & b(0) \\ -a^*(1) & a^*(0) & 0 & a(2) \\ -a^*(2) & 0 & a^*(0) & -a(1) \\ 0 & -a^*(2) & a^*(1) & a(0) \end{bmatrix}$$

Note that $\mathbf{X}_{\mathbf{a}}$ in this example is the well-known *Octonion* orthogonal design [11]. It is interesting to note that adding the symbol $b(0)$ does not affect the diversity order achieved by Layer $\mathcal{A}$ symbols while increasing the overall rate of the code. The same rate-diversity tuple is achieved if $b(0)$ is placed on any other position along the main anti-diagonal of $\mathbf{X}_{\mathbf{a},\mathbf{b}}$. Due to space limitations, we refer the interested reader to [10], [12] for other constellation-independent code constructions and proofs of their achievable rate-diversity tuples.

*2) Constellation-Dependent Constructions:* In this second class of constructions, we scale the information symbols in Layer $\mathcal{B}$ only by a real scalar $\frac{1}{K}$ (where $K > 1$) designed to guarantee the desirable rate-diversity tuple while optimizing the coding gain for Layer $\mathcal{A}$ or $\mathcal{B}$ to meet QoS requirements. As an example, for a unit-radius M-PSK constellation, our proposed power scaling scheme results in a transmitted signal constellation which consists of two concentric circles of radii 1 and $\frac{1}{K}$. This increases the peak-to-minimum power ratio (PMPR) to $K^2 > 1$ compared to a conventional M-PSK constellation with PMPR=1. For the considered case of 2 diversity layers each using an M-PSK constellation, $K$ is designed to achieve a tradeoff between maximizing the ratio of available coding gain ($CG_{\mathcal{A}}$ or $CG_{\mathcal{B}}$) and the transmitted signal PMPR for the 2 diversity layers. Since the achievable coding gain for each layer depends on the signal constellation size $M = |\mathcal{S}|$, the optimum $K$ will also be a function of $M$ and can be computed off-line. Next, we present 3 examples from this class of constellation-dependent constructions. The first and third examples transmit 2 diversity layers using 2 and 4 antennas, respectively, while the second example transmits 3 diversity layers using 3 antennas.

**Example 2:** Here Layer $\mathcal{A}$ contains 1 information symbol $\{a(0)\} \in \mathcal{S}$ and Layer $\mathcal{B}$ contains 2 information symbols $\{b(0), b(1)\} \in \mathcal{S}$. This example achieves the tuple $(\frac{1}{2}\log_2|\mathcal{S}|, 2M_r, \log_2|\mathcal{S}|, M_r)$.

$$\mathbf{X}_{\mathbf{a},\mathbf{b}} = \mathbf{X}_{\mathbf{a}} + \frac{1}{K}\mathbf{X}_{\mathbf{b}} = \begin{bmatrix} a(0) & \frac{b(0)}{K} \\ -\frac{b^*(1)}{K} & a^*(0) \end{bmatrix}$$

For QPSK, we showed in [9] that $K(>1)$ can be any positive real number *not* equal to $\sqrt{2}$. The diversity for Layer $\mathcal{B}$, namely $D_b = M_r$ is apparent from the code design.

Figure 1 depicts the BER performance of Example 2 for $K = \sqrt{1.5}$ and $K = \sqrt{3}$ (lower and higher than the forbidden value $K = \sqrt{2}$) where the higher value of $K = \sqrt{3}$ improves the coding gain of Layer $\mathcal{A}$ while it reduces that of Layer $\mathcal{B}$ and vice versa for $K = \sqrt{1.5}$.

**Example 3 :** In this example, $M_t = 3$, $T = 4$ symbol periods, and we have 3 diversity layers. Here, Layer $\mathcal{A}$ contains 3 information symbols $\{a(0), a(1), a(2)\} \in \mathcal{S}$, Layer $\mathcal{B}$ contains 1 information symbol $b(0) \in \mathcal{S}$ and so does Layer $\mathcal{C}$ with $c(0) \in \mathcal{S}$. Therefore, this code achieves a total rate of $R = R_a + R_b + R_c = \frac{5}{4}\log_2|\mathcal{S}|$.

$$\mathbf{X}_{\mathbf{a},\mathbf{b},\mathbf{c}} = \mathbf{X}_{\mathbf{a}} + \frac{1}{K}\mathbf{X}_{\mathbf{b}} + \frac{1}{K}\mathbf{X}_{\mathbf{c}} = \begin{bmatrix} a(0) & -a^*(1) & -a^*(2) & \frac{c^*(0)}{K} \\ a(1) & a^*(0) & \frac{b^*(0)}{K} & -a^*(2) \\ a(2) & \frac{b^*(0)}{K} & a^*(0) & a^*(1) \end{bmatrix}$$

With a proper choice of $K$, this code was shown in [9] to achieve the rate-diversity tuple $(\frac{3}{4}\log_2|\mathcal{S}|, 3M_r, \frac{1}{4}\log_2|\mathcal{S}|, 2M_r, \frac{1}{4}\log_2|\mathcal{S}|, 1M_r)$. For a QPSK constellation, $K$ can be any real number greater than 1 and *not* equal to $\sqrt{2}$ (see [9] for justification). Similarly, we can easily enumerate the permissible values of $K$ for higher-order constellations. As in Example 2, the choice of $K$ represents a tradeoff between the achievable coding gains of the 3 diversity layers and we selected $K = 1.6$ for QPSK in [9] to achieve a good coding gain compromise between the layers at a controllable PMPR.

**Example 4:** In this example, $M_t = 4$ and $T = 4$ symbol periods. Here, Layer $\mathcal{A}$ contains 3 information symbols $\{a(0), a(1), a(2)\} \in \mathcal{S}$ and Layer $\mathcal{B}$ contains 2 information symbols $\{b(0), b(1)\} \in \mathcal{S}$. This code has a total rate of $R = R_a + R_b = \frac{5}{4}\log_2|\mathcal{S}|$ and was shown in [9] to achieve the tuple $(\frac{3}{4}\log_2|\mathcal{S}|, 4M_r, \frac{1}{2}\log_2|\mathcal{S}|, 2M_r)$.

$$\mathbf{X}_{\mathbf{a},\mathbf{b}} = \mathbf{X}_{\mathbf{a}} + \frac{1}{K}\mathbf{X}_{\mathbf{b}} = \begin{bmatrix} a(0) & -a^*(1) & -a^*(2) & \frac{b(1)}{K} \\ a(1) & a^*(0) & \frac{b^*(0)}{K} & -a^*(2) \\ a(2) & \frac{b^*(0)}{K} & a^*(0) & a^*(1) \\ \frac{b(1)}{K} & a(2) & -a(1) & a(0) \end{bmatrix}$$

Furthermore, it was shown in [9] that, assuming a QPSK constellation for both diversity layers, $K = \sqrt{3}$ is a good design choice for this example. The BER performance of Examples 3 and 4 is given in [9].

## III. Transceiver Design Issues

In this section, we investigate two LDE-STBC transceiver design issues; namely reduced-complexity coherent decoding and integration with hierarchical modulation to vary the coding gain (in addition to varying the diversity order). Differential encoding/decoding of LDE-STBC is investigated in [20].

### A. Reduced-Complexity Coherent Decoding

Coherent decoding assumes that CSI is estimated at the receiver through training. As shown in [13], for training-based least-squares channel estimation, the lowest channel estimation error variance is achieved using orthogonal codewords for training such as orthogonal designs [24]. Due to their linearity over the complex field, all of the LDE-STBC examples presented in Section II are amenable to computationally-efficient lattice decoding strategies, such as the *sphere decoder* [7]. Alternatively, decoding can also be performed using successive interference cancellation (IC) where the more reliable high-diversity layer is decoded first and its effect on the received signal is cancelled followed by decoding of the lower-diversity layer. It is well-known that successive IC can suffer significant performance degradation due to error propagation effects. Its performance can be improved by using a *hybrid* ML/IC algorithm which performs a full ML search on a selected subset of the information symbols and for each ML search candidate, its interfering effect is cancelled from the received signal followed by matched-filtering-based decoding of the remaining information symbols.

The main novelty when applying this scheme to decode our LDE-STBC constructions is to carefully select the information symbols in the ML search so that when their interfering effect is cancelled, the equivalent channel matrix for the remaining information symbols is *orthogonal* making matched-filter decoding optimal for them while reducing decoding complexity significantly compared to full ML search (see [9] for more details).

### *B. Varying the Coding Gain*

The error rate of a space-time coding scheme is characterized by both diversity level and coding gain [23]. Our LDE-STBC designs provide UEP by assigning different diversity levels to the different information layers. For a more flexible design, it is also desirable for LDE-STBC to provide variable coding gains. The constellation-dependent constructions (Examples 2-4) can achieve a tradeoff in the coding gains of the different diversity layers by varying the power scaling factor $K$, as shown in Section II. However, all information bits within each diversity layer achieve the same coding gain. A more flexible approach to achieve a variable coding gain across the information bits within each diversity layer is realized by integrating LDE-STBC with hierarchical modulation [25] as described in [15]. This allows the system designer to increase the number of available QoS classes (through variable coding gain) without modifying the LDE-STBC design and without trading off the achievable coding gain across the diversity layers as in Code Examples 2-4.

**Remarks**

- Hierarchical modulation uses non-uniform constellations to provide greater Euclidean distance between the more important bits compared to the less important bits. Therefore, different bit positions of a symbol achieve variable coding gains and thus UEP is achieved at the *bit level*. This is in contrast with LDE-STBC which transmits information symbols in layers each at a different diversity level; hence, providing UEP at the *symbol level*. Nevertheless, as shown in [15], the 2 schemes can be integrated to provide various UEP levels by varying the diversity orders and coding gains at both the bit and symbol levels, as desired by the system designer.

- The focus of this paper is on flat-fading channels. However, LDE-STBC can be implemented over frequency-selective channels by carefully integrating it with equalization schemes such as OFDM. Such a scheme was investigated in [26] where Code Examples 2-4 were implemented over adjacent OFDM blocks (over which the channel is assumed fixed) and the power scaling factor $K$ was designed to achieve a practical tradeoff between coding gain and peak-to-average ratio (PAR) of the transmitted OFDM signal while providing robustness against carrier frequency offset.

## IV. Application to Wireless Multicasting

In wireless multicasting [4], information streams with multiple QoS requirements are transmitted to multiple receivers simultaneously. The inherent broadcast nature of wireless transmission is thus exploited in multicasting to reduce the traffic load compared to conventional point-to-point communications. Depending on factors such as receiver sensitivity, decoding algorithms, and number of receive antennas, the intended receivers of a multicasting group can be classified into less-capable or more capable receivers. Less-capable receivers require a higher SNR to decode the same message compared to more-capable receivers; hence, limiting the achievable throughput. Therefore, the superior decoding ability of more-capable receivers remains under-utilized, leaving room to improve overall system throughput. This can be achieved by splitting each information symbol into basic and additional (enhancement) bits so that each receiver in the coverage area can decode the basic bits but only more-capable receivers are able to decode the additional bits [5], [3]. An efficient technique to realize such splitting is by integrating hierarchical modulation [25] with LDE-STBC to provide variable coding gains among the bits of a symbol in addition to variable diversity levels offering more flexibility to multicasting applications. Such a scheme provides an efficient mechanism to adaptively control the coverage regions of information streams belonging to different QoS classes by varying the constellation separation angles without changing the transmitter power level.

**Simulation Results**

For simplicity, we use the same hierarchical signal constellations (one of the two shown in Figure 2) in both transmission layers. Of the two bits required to uniquely represent the QPSK symbols, we consider the most-significant bit (MSB) and least-significant bit (LSB) as basic and additional messages, respectively. Figure 3 depicts the BER of Layer $\mathcal{A}$ of Code Example 1. It is clear that at a specified BER, the SNR gap between the MSB and LSB varies depending on the constellation separation angles. Compared with the uniform QPSK constellation, the MSB is protected by a higher coding gain at the cost of a reduced coding gain for the LSB. It is noteworthy that all BER curves exhibit the same full diversity order of $4$ (this is verified by the identical slopes of the BER curves). Similar conclusions can be drawn for Layer $\mathcal{B}$ where a diversity order of $1$ is achieved [15].

In Code Example $1$, we introduce an additional message and use hierarchical QPSK modulation to increase the data rate. Therefore, for a fixed transmitted power, the coverage area of the basic message will shrink compared to the case when only a basic message is transmitted as in the Octonion OSTBC. But now we have an additional message in some portion of the cell coverage area. Furthermore, depending on the hierarchical QPSK constellation separation angles, there is a tradeoff in the coverage regions between more-important and less-important bits of each QoS class. To measure the change in the coverage area (expansion or shrinkage), we calculate the coverage area normalized to the corresponding Octonion OSTBC and present the result in Fig. 4. For a fair comparison at the same spectral efficiency of 3 bits PCU, 16-PSK modulation is used for the Octonion OSTBC while Code Example $1$ is hierarchically 8-PSK modulated. We assume a log-distance path-loss fading model with loss exponent $4$ and target BERs of $10^{-4}$ and $10^{-2}$ for Layers $\mathcal{A}$ and $\mathcal{B}$, respectively. It is evident from Fig. 4 that for carefully-designed constellations, the basic messages cover more area than the baseline Octonion code, i.e. the normalized coverage ratio is greater than $1$. For example, as demonstrated in Fig. 4, at $7.5^o$ separation angle, if the Octonion 16-PSK OSTBC covers an area of $1$ square kilometer, the basic and additional messages of Layer $\mathcal{A}$ will cover over $2.5$ and $0.7$ square kilometers, respectively. In addition, we showed in [15] that the basic and additional messages of the lower-diversity Layer $\mathcal{B}$ cover $0.5$ and $0.18$ square kilometers, respectively, for the same constellation separation angle.

Finally, using Code Example $1$ with hierarchical $8$-PSK modulation achieves data rates of $1.5$ Mbps and $0.75$ Mbps for the basic and additional messages of Layer $\mathcal{A}$, respectively, and corresponding rates of $0.4$ Mbps and $0.23$ Mbps for Layer $\mathcal{B}$ for a total rate of 2.93 Mbps. This is to be compared to a total rate of 2.25 Mbps for the Octonion OSTBC (i.e. 30 $\%$ increase) split as $1.5$ Mbps and $0.75$ Mbps for the basic and additional messages, respectively. For these data rate calculations, we assumed a dedicated channel bandwidth of 1 MHz, 20 codewords per data-frame and target frame error rates of $10^{-3}$ and $10^{-1}$ for Layers $\mathcal{A}$ and $\mathcal{B}$, respectively.

## V. Conclusions

In this paper, we investigated the construction, transceiver design, performance, and applications of linear diversity-embedding space-time block codes. Under constructions, we discussed both constellation-independent and dependent code examples for 2 and 3 diversity layers and 2-4 transmit antennas. For the constellation-dependent constructions, we showed how to allocate the transmit power to the diversity layers to meet the target QoS levels while minimizing the peak-to-minimum transmit power ratio. Under transceiver design, we presented a reduced-complexity coherent ML decoding algorithm which exploits the algebraic structure of LDE-STBC. Furthermore, we discussed how to vary the coding gain both across and within the diversity layers. Under performance analysis, we demonstrated the UEP capabilities of LDE-STBC in provisioning variable diversity orders and coding gains and the resulting performance advantages over conventional equal-error-protection single-layer STBC. Finally, under applications, we considered wireless multicasting and quantified the performance gains achieved by our codes in terms of increased coverage area.

**Acknowledgement**

The authors would like to thank J. Chui, S. Das, S. Dusad, H. Minn, D. Wang for several interesting discussions and contributions on the subject of this paper.

## REFERENCES

[1] S. M. Alamouti,"A Simple Transmit Diversity Technique for Wireless Communications", *IEEE J. Select. Areas Commun*, pages 1451-1458, October 1998.

[2] N. Al-Dhahir, "Overview and Comparison of Equalization Schemes for Space-Time-Coded Signals with Application to EDGE", *IEEE Transactions on Signal Processing*, pages 2477-2488, October 2002.

[3] A. R. Calderbank and N. Seshadri, "Multilevel Codes for Unequal Error Protection", *IEEE Transactions on Information Theory*, pages 1234-1248, July 1993.

[4] P. Chaporkar and S. Sarkar, "Wireless Multicast: Theory and Approaches", *IEEE Transactions on Information Theory*, pages 1954-1972, June 2005.

[5] T. M. Cover, "Broadcast Channels", *IEEE Transactions on Information Theory*, pages 2-14, January 1972.

[6] R.V. Cox, J. Hagenauer, N. Seshadri and C.-E.W. Sundberg, "Subband Speech Coding and Matched Convolutional Channel Coding for Mobile Radio Channels", *IEEE Transactions on Signal Processing* pages 1717-1731,August 1991.

[7] M. O. Damen, A. Chkeif, and J.-C. Belfiore, "Lattice Codes Decoder for Space-Time Codes", *IEEE Communication Letters*, pages 161-163, May 2000.

[8] S. Das, N. Al-Dhahir, S. Diggavi, R. Calderbank,"Opportunistic Space-Time Block Codes", *IEEE Vehicular Technology Conference*, pages 2025-2029, September 2005.

[9] S. Das and N. Al-Dhahir, "New Diversity-Embedding STBC Constructions", *In IEEE Workshop on Signal Processing Advances in Wireless Communication SPAWC 2006*, pages 1-5, June 2006.

[10] S.N. Diggavi, N. Al-Dhahir and A. R. Calderbank, "Diversity-Embedded Space-Time Codes" *Global Telecommunications Conference*, pages 1909-1914, December 2003.

[11] S.N. Diggavi, N. Al-Dhahir, A. Stamoulis, A.R. Calderbank, "Great Expectations : The Values of Spatial Diversity in Wireless Networks", Proceedings of the IEEE (Special Issue on Gigabit Wireless), pages 219-270, February 2004

[12] S.N.Diggavi, A.R.Calderbank, S.Dusad, N.Al-Dhahir, "Diversity-Embedded Space-Time Codes", IEEE Transactions on Information Theory, pages 33-50, January 2008.

[13] C. Fragouli, N. Al-Dhahir, and W. Turin, "Training-Based Channel Estimation for Multiple-Antenna Broadband Transmissions", *IEEE Transactions on Wireless Communications*, pages 384-391, March 2003.

[14] J. Hagenauer, ”Rate Compatable Punctured Convolutional Codes and Their Applications”, *IEEE Transactions on Communications*, pages 389-400, April 1988.

[15] K.M.Z. Islam and N. Al-Dhahir, ”Hierarchical Diversity-Embedding Space-Time Block Coding”, *Asilomar Conference on Signals, Systems, and Computers*, pages 1274-1278, November 2006.

[16] E. Larsson and W. Wong ”Nonuniform Unitary Space-Time Codes for Layered Source Coding”, *IEEE Transactions on Wireless Communications*, pages 958-965, May 2004.

[17] S. Lin, A. Stefanov, Y. Wang, ”Joint Source and Space-Time Block Coding for MIMO Video Communications”, *Vehicular Technology Conference*, pages 2508-2512, September 2004.

[18] H. F. Lu and P. V. Kumar. ”Rate-Diversity Tradeoff of Space-Time Codes with Fixed Alphabet and Optimal Constructions for PSK Modulation”, *IEEE Transactions on Information Theory*, pages 2747-2752, October 2003.

[19] B. Masnick and J. Wolf, ”On Linear Unequal Error Protection Codes”, *IEEE Transactions on Information Theory*, pages 600-607, October 1967.

[20] P. Rabiei and N. Al-Dhahir, ”Differential Diversity-Embedding Space-Time Block Coding”, *Asilomar Conference on Signals, Systems, and Computers*, pages 1291-1295, November 2006.

[21] N. Rahnavard, H. Pishro-Nik, and F. Fekri, ”Unequal Error Protection Using Partially Regular LDPC Codes”, *IEEE Transactions on Communications*, pages 387-391, March 2007.

[22] M. Sabir, R. Heath, A. Bovik, ”An Unequal Error Protection Scheme for Multiple Input Multiple Output Systems”, *Asilomar Conference n Signals, Systems, and Computers*, pages 575-579, November 2002.

[23] V. Tarokh, N. Seshadri and A. R. Calderbank,”Space-Time Codes for High Data Rate Wireless Communication: Performance Criterion and Code Construction”, *IEEE Transactions on Information Theory*, pages 744-765, March 1998.

[24] V. Tarokh, H. Jafarkhani, and A.R. Calderbank, ”Space-Time Block Codes from Orthogonal Designs”, *IEEE Transactions on Information Theory*, pages 1456-1467, July 1999.

[25] P. K. Vitthaladevuni and M. Alouini,”Exact BER Computation of Generalized Hierarchical Constellations”, *IEEE Transactions on Communnications*, pages 2030-2037, December 2003.

[26] D. Wang, H. Minn, and N. Al-Dhahir, ”An Opportunistic STBC-OFDM Scheme with Reduced PAR in the Presence of Frequency Offset”, *IEEE International Conference on Acoustics, Speech, and Signal Processing*, pages 369-372, April 2007.

[27] L. Zheng and D. Tse, ”Diversity and Multiplexing: A Fundamental Tradeoff in Multiple Antenna Channels”, *IEEE Transactions on Information Theory*, pages 1073-1096, May 2003.

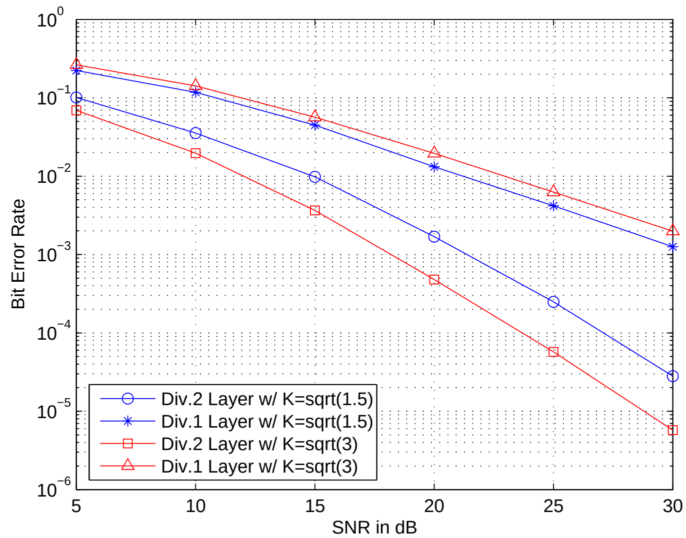


Fig. 1. Performance of Example 2 with ML Decoding

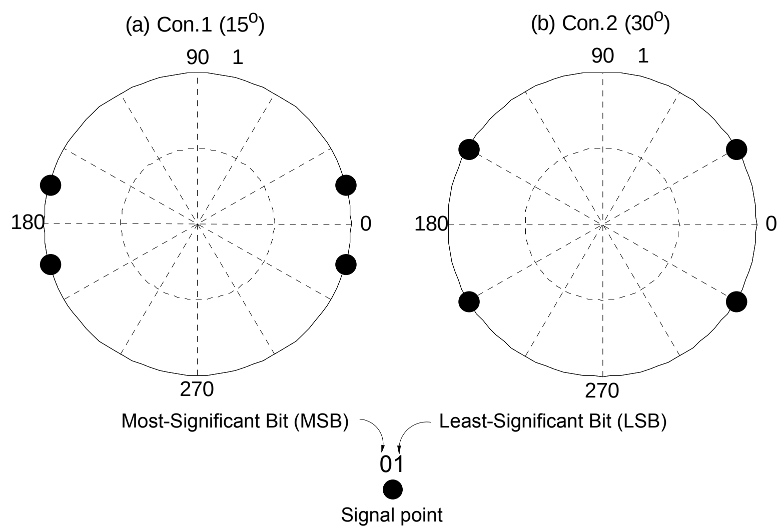


Fig. 2. Two Hierarchical QPSK Constellation Examples

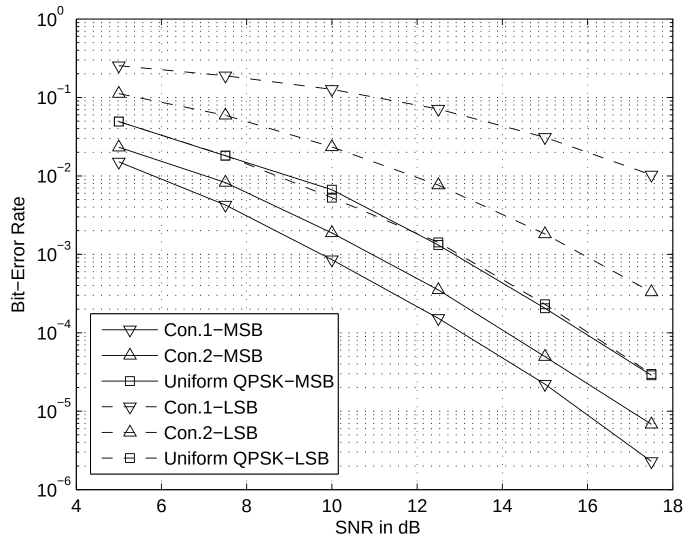


Fig. 3. Layer $\mathcal{A}$ BER Performance for Example 1 with Hierarchical and Uniform QPSK Constellations

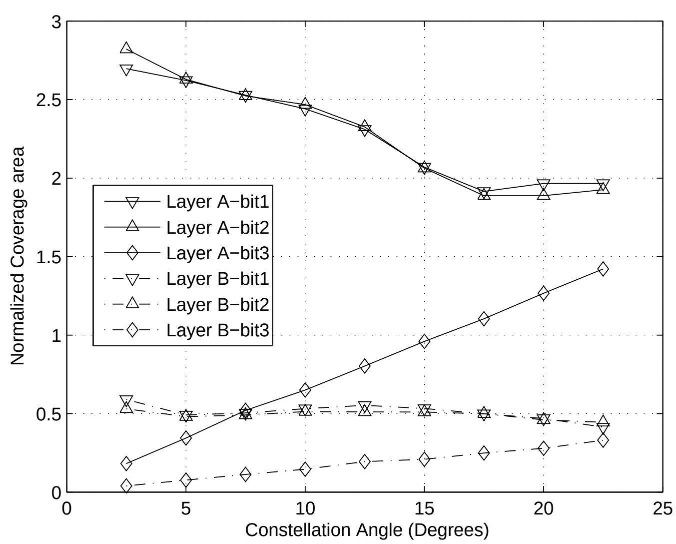


Fig. 4. Normalized Coverage Area of Example 1 with 8-PSK Hierarchical Modulation with respect to 16-PSK Octonion at Spectral Efficiency of 3 bits PCU